

Impedance Matching

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The term **Impedance** is a general term which can be applied to any electrical ethnicity which impedes (obstruct) the flow of the current. [Oliver Heaviside](#) (1850-1925) coined **Impedance** definition, as well as many other terms of art in electromagnetic theory: Inductance, Admittance, Conductance, Permeability, Permittance (Susceptance), Reluctance, Electret.

Oliver Heaviside also reformulated Maxwell's field equations in terms of electric and magnetic forces and energy flux, and independently co-formulated **Vector Analysis**, which is needed to express the **impedance** in coordinates.

Impedance Matching was originally developed for electrical power, but can be applied to any other field where a form of energy (not necessarily electrical) is transferred between a source and a load.

- The first **Impedance Matching** concept in RF domain was related to **Antenna Matching**. Designing an antenna can be seen as matching the free space to a transmitter or to a receiver.

Impedance Matching is always performed between two specified terminations

- The main purpose of Impedance Matching is to match two different terminations (R_{source} and R_{Load}) through a specific pass-band, without having control over stop-band frequencies. We may assume that component losses are negligible but parasitic effects need to be considered.

Reactance and LC Resonance

- Reactance **X** is a measure of the opposition to the current of Capacitance **C** and Inductance **L**. Reactance is measured in ohms and varies with the frequency of the AC signal.
- Reactance takes two forms: Inductive (X_L), and Capacitive (X_C).

$$X_L = \omega L = 2 \pi f L$$

$$X_C = 1 / (\omega C) = 1 / (2 \pi f C)$$

Where: ω is angular frequency, f is frequency, L is inductance, and C is capacitance.

- When the magnitudes of L reactance and C reactance are equal, the L-C pair resonates.
- At resonance the net reactance of a series-connected L-C circuit is zero (a short circuit), and the net reactance of a parallel-connected L-C circuit is infinite (an open circuit).
- The resonant frequency is getting by equating the magnitudes of the **L** and **C** reactances ($X_L = X_C$)

Resonant frequency is given by:
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Quality-Factor

The **Quality Factor Q** serves as a measure of a reactance's purity (how close it is to being a pure reactance, and not a resistance), and is defined as the ratio of the energy stored in a component to the energy dissipated by the component.

$$Q = \frac{2\pi \cdot (\text{max instantaneous energy stored})}{\text{energy dissipative per cycle}}$$

- **Q** is a dimensionless unit and is expressed function of reactance **X** and resistance **R** as:
$$Q = X / R$$
- It should be stated that **Q** of the L-C circuit is defined at circuit resonance.
- If the circuit reactance is plotted as a function of frequency, the slope of the reactance at resonance is a measure of **Q**.

- The Quality Factor **Q** of a reactive component (inductor or a capacitor) is the ratio of its reactance magnitude to its resistance.

Q of an inductor: $Q = \frac{X_L}{R_L} = \frac{\omega L}{R_L}$

Q of a capacitor: $Q = \frac{X_C}{R_C} = \frac{1}{\omega C R_C}$

Unloaded-Q

- The loaded quality factor **Q_L** is the ratio of the magnitude of the reactance of either L or C at resonance, to the total circuit resistance.
- In real physical reactive elements, there are always some resistive losses.
- The loss in a component (a capacitor or an inductor) can be described in terms of its Q.

For example, if a lossy inductor is placed in parallel with a lossless capacitor, the Q of the resulting parallel circuit is almost equal to the Q of the inductor.

- The **unloaded-Q (Q_u)** is the Q associated with the reactive elements only (i.e., without the load).

$$\frac{1}{Q_u} = \frac{1}{Q_{ind}} + \frac{1}{Q_{cap}}$$

Loaded-Q

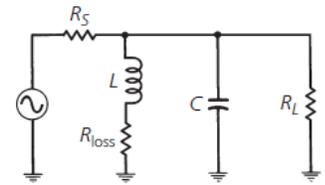
The **loaded-Q** of a resonant circuit is dependent upon three main factors:

1. The source resistance (**R_s**).
2. The load resistance (**R_L**).
3. The component Q-factor as defined above.

The resonant circuit sees an equivalent **R_P** resistance, **R_s** in parallel with **R_L**, as

The **R_P** resistance is smaller in value than either **R_s** or **R_L**

$$R_P = (R_s \times R_L) / (R_s + R_L)$$



Assuming lossless components: $Q = \frac{R_P}{X_P}$

R_P = equivalent parallel resistance of **R_s** and **R_L**
X_P = inductive or capacitive reactance (both equal at resonant frequency)

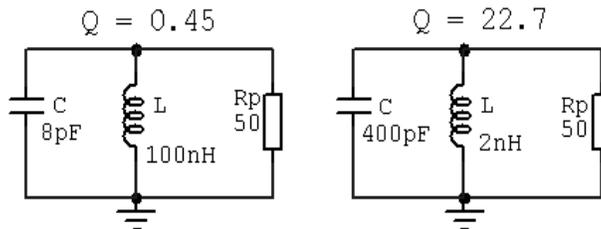
- To optimize the loaded-Q there are two options. First, we can select the optimum values for source and load impedances, or second, we can use different ratio values for L and C.
- Decreasing the **R_P** will decrease the Q of the resonant circuit, and an increase in **R_P** will increase the Q of the circuit. Can get the same effect if we keep **R_P** constant and varying **X_P**.

Example:

Using a parallel L-C circuit, at the same resonant frequency (178MHz), we get different Q values just changing the L and C values (ratio), when keeping **R_P** constant (50 Ω).

- If **R_P** = 50 Ω, C = 8pF, L = 100nH, at 178MHz we get: **X_P** = **X_C** = **X_L** = 111.8 Ω, results Q = 0.45

- If **R_P** = 50 Ω, C = 400pF, L = 2nH, at 178MHz we get: **X_P** = **X_C** = **X_L** = 2.2 Ω, results Q = 22.7

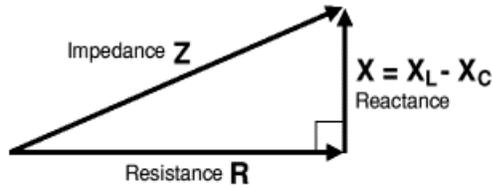


Impedance

Impedance is an important parameter to characterize electronic circuits, components, and materials used to make components.

- Impedance **Z** is defined as the total opposition of a device or circuit to the flow of an alternating current (AC) at a given frequency, and is represented as a complex quantity which is graphically shown on a vector plane.

- An impedance vector consists of a real part (resistance, R) and an imaginary part (reactance, X).
- Impedance can be expressed using the rectangular-coordinate form: $Z = R + jX$ or in the polar form as a magnitude and phase angle: $Z = |Z| \angle \theta$



$$\text{Impedance, } Z = \sqrt{R^2 + X^2}$$

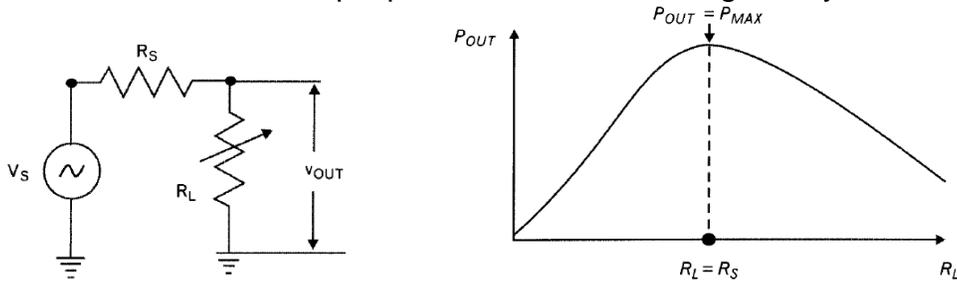
- Impedance varies with frequency, when the effect of resistance is constant regardless of frequency.

Maximum Power Transfer

Designing circuits involves the efficient transfer of the signals.

In the early days of electric motors, it was found that to get the most efficient transfer of power from the battery (source) into the motor (load) required that the resistance of the different parts of the circuit be the same, in other words, matched; this is known as the **maximum power transfer theorem**.

- For DC circuits, maximum power will be transferred from a source to its load if the load resistance equals the source resistance. A simple proof of this theorem is given by the following example:



The circuit and graph to prove the condition for maximum power transfer

$$v_{OUT} = \frac{R_L}{R_S + R_L} (v_S)$$

$$P_{OUT} = \frac{v_{OUT}^2}{R_L} = \frac{\left(\frac{R_L}{R_S + R_L}\right)^2 (v_S^2)}{R_L}$$

$$= \frac{R_L}{(R_S + R_L)^2} (v_S^2)$$

(v_S and v_{out} refer to r.m.s. value)

With the equation above we can calculate and plot the power delivered to the load for various load resistances. When the load is zero, no power can be delivered to it, and as the resistance R_L increases above zero, the voltage across it and dissipated power increases.

As the R_L further increases, the value of P_{OUT} reaches a peak (where $R_L = R_s$) and thereafter decreases.

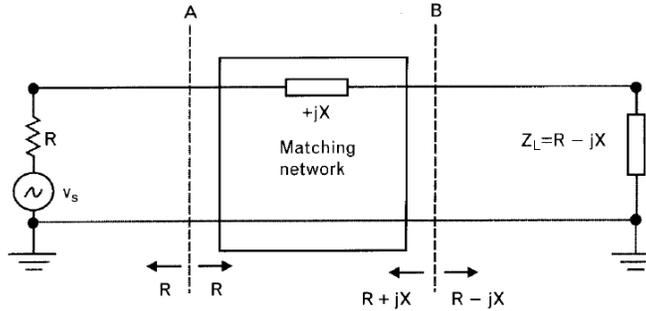
Impedance Matching

The main role in any Impedance Matching scheme is to force a load impedance to “look like” the complex conjugate of the source impedance, and maximum power can be transferred to the load.

- When a source termination is matched to a load with passive lossless two-port network, the source is conjugated matched to the input of the network, and also the load is conjugate matched to the output of the network.
- Any reactance between R_s and R_L reduces the current in R_L and with it the power dissipated in R_L .

To restore the dissipation to the maximum that occurs when $R_s = R_L$, the net reactance of the loop must be zero. This occurs when the load and source are made to be complex conjugates one of another, so they have the same real parts and opposite type reactive parts.

If the source impedance is $Z_s = R + jX$, then its complex conjugate would be $Z_s^* = R - jX$.



Impedance Matching of a resistive source and a complex load for maximum power transfer

- Using only one series reactive element between two equal resistive terminations creates a voltage drop that reduces the voltage across the load. Impedance Matching can eliminate or minimize the unwanted reactance through a range of frequencies.
- The matching process becomes more difficult when real parts of the terminations are unequal, or when they have complex impedances.

Example:

Match a 50Ω resistive source at 100MHz, to a 50Ω resistive load that has in series a 1.59pF capacitance.

Since the terminations are equal, the required matching circuit can be a series inductor to negate the reactance of the series capacitance.

$$X_C = 1 / (\omega \times C) = 1 / (2 \pi \times f \times C) = 1 / (2 \times 3.14 \times 100 \times 10^6 \times 1.59 \times 10^{-12}) = 1000 \Omega$$

At 100 MHz the capacitive reactance $X_C = -j1000 \Omega$

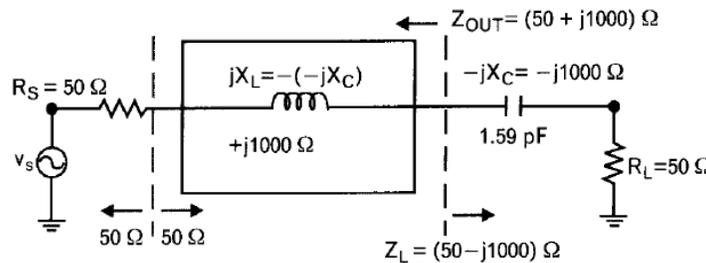
We need to negate this capacitive reactance by adding $+j1000 \Omega$ in series (series inductor).

At resonance: $X_C = X_L = \omega \times L$

The matching inductor: $L = X_L / \omega = 1000 / (2 \times \pi \times f) = 1000 / (2 \times 3.14 \times 10 \times 10^6) = 1.59\mu\text{H}$

Adding just a $1.59\mu\text{H}$ series inductor between the two resistive terminations provides match at 100MHz.

- Conjugate match is obtained only at one frequency, where the two reactive components resonate. As the frequency is increased or decreased from this value, the transmitted power rolls off at a rate determined by the loaded-Q of the circuit.



When $R_s = R_L$ matching requires only a series reactive element

The 3-dB frequency bandwidth of a matching network is: $BW_{3dB} = f_R / Q_{series}$

Q for series resonance is:

$Q_{series} = (\text{Reactance of one resonant element at } f_R) / (\text{Total series resistive loading})$

In the above example using 1000Ω reactance, and $(50+50)\Omega$ total resistive loading, gives $Q_{series} = 10$

So, $BW_{3dB} = (10 \times 10^6) / 10 = 10\text{MHz}$

If in the above example the load capacitance was in parallel with R_L , then a parallel inductor would be used to resonate the load parasitic capacitance.

In this case Q for parallel resonance is:

$$Q_{\text{parallel}} = (\text{Total parallel resistive loading}) / (\text{Reactance of one resonant element at } f_R)$$

- Matching high-Q terminations leads to narrow bandwidths.

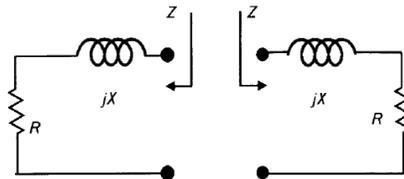
For a given resistance, there are few possibilities for high-Q when associated parasitic reactance is one of the following:

- Large series or small parallel inductance
- Small series or large parallel capacitance

When the resistive portion is very small or very large (i.e. input resistance of a high-power transistor, or output impedance of a low-current device), even small amount of parasitic inductance or capacitance can lead to high-Q making Impedance Matching a challenging task.

- In parallel resonant circuits, high-value loading resistors lead to high-Q, while in series circuits the opposite is true.
- A broader band match usually can be obtained when the tuning is performed close to the load. Accepting an approximate match at the center frequency may result in a better average match over the operating band.

Common mistake of Impedance Matching is to connect a source $Z_s = R + jX$ to a load $Z_L = R + jX$. The net result is the real part of the source, R, sees an effective load of $R + j2X$, leading to power reflection back to the source.

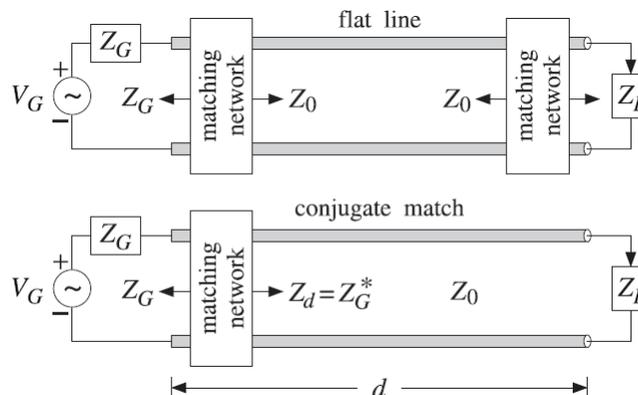


Interconnecting two identical complex impedances doesn't lead to maximum power transfer

If the two terminations are not equal (as in the previous example) first have to develop a technique to take care about this issue, and then expand the technique to also include reactive elements.

At a single frequency, an appropriate pair of L-C elements can provide a match to both terminations.

- However, **conjugate matching** is not the same as **reflectionless matching**, which refers to matching the load to the line impedance, $Z_L = Z_0$, in order to prevent reflections from the load. In practice, we must use matching networks at one or both ends of the transmission line to achieve the desired type of matching. Figure below shows the two typical situations that arise.



Reflectionless and Conjugate matching of a transmission line

- In the first picture, referred to as a flat line, both the generator and the load are matched, so that effectively, the impedances of the generator, of the load and of the transmission line, all are equal, $Z_G = Z_L = Z_0$. There are no reflected waves and the generator (which is typically designed to operate into Z_0) transmits maximum power to the load, as compared to the case when $Z_G=Z_0$ but $Z_L \neq Z_0$.
- In the second case, the load is connected to the line without a matching circuit and the generator is conjugate-matched to the input impedance of the line, that is, $Z_d = Z_G^*$. The line remains conjugate matched everywhere along its length, and therefore, the matching network can be inserted at any convenient point, not necessarily at the line input.
- Because the value of Z_d depends on Z_L and the frequency ω , the conjugate match will work as designed only at a single frequency.
- On the other hand, if the load and generator are purely resistive and are matched individually to the line, the matching will remain reflectionless over a larger frequency bandwidth.

Conjugate matching is usually accomplished using L-section reactive networks.

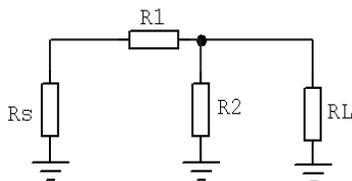
Reflectionless matching is achieved by essentially the same methods as antireflection coating.

Pros and Cons of Complex Conjugate impedance matching in active RF circuits

- In an active RF circuit, **complex conjugate matching** produces the maximum small signal transfer of power from a source (that is not a transmission line) to a load, at the sacrifice of:
 - Stability (lack of self-oscillation)
 - Bandwidth
 - Noise
 - Harmonic distortion
 - Intermodulation distortion
 - Temperature coefficients
 - Large signal power
 - Slew rate
 - Large signal power conversion efficiency
 - Amplitude frequency response
 - Phase frequency response
 - Differential group delay distortion
 - Pulse fidelity
 - Overload recovery
 - Sensitivity to transistor production lot parameter variations
- If the source impedance cannot be changed, making the source impedance complex conjugately match the load impedance, this will degrade the gain of the circuit by 6dB and reduces the signal power conversion efficiency a further compounded 50%.

Impedance Matching using Resistor Networks

Using a resistive network can match simultaneous input and output, **but create more loss**.



$$R1 = R_s - (R_L \times R2) / (R_L + R2)$$

$$R2 = R_L - \sqrt{[(R_s) / (R_s - R_L)]}$$

For example, if there is a series-parallel resistor combination to match 50 Ω source and 5 Ω load simultaneous, the transmission loss it will increase to 21.6dB.

Impedance Matching using Transformers

Another possible option to match resistive source and resistive load is to use transformers.

- Transformers convert source power from one voltage and current level to another voltage and current level.
- The load impedance is transformed as a square of the voltage-transformation ratio.

The ratio of the voltage transformation comes from the number of turns on the input winding (primary), divided by the number of turns on the output winding (secondary).

- To achieve Impedance Matching, the Turns Ratio of the transformer is the square root of the ratio of Load Resistance over the Source Resistance.

$$\text{Turns Ratio} = \sqrt{\frac{\text{Load Resistance}}{\text{Source Resistance}}}$$

- For example, to match a 50 Ω resistive source to a 5 Ω resistive load we need a transformer with:

$$\text{Turns Ratio} = \sqrt{\frac{5}{50}} = 0.32$$

For example, if the primary has 100 turns, the secondary must have $100 \times 0.32 = 32$ turns.

The terms step-up or step-down refer to ability of the transformer to change (transform) the voltage or the current that passes through it.

- The amount of power ($P = I \times V$) that goes into a transformer is always equal to the amount of power that comes out (discounting negligible losses). While a step-down transformer is changing the input voltage to a lower voltage, it is also changing the input current to higher current.
- Transformers match only the “real” part of the impedance. If there is a large amount of reactance in the load, a transformer will not eliminate these reactive components. In fact, a transformer may exaggerate the reactive portion of the load impedance. This reactive component results in power that is reflected to the generator.
- Transformers however works poorly at microwave frequencies, but provide wider bandwidths than L-C matching circuits.

Advantages of using transformers for Impedance Matching are:

- Wide bandwidth, exceeding 1000 MHz.
- Excellent amplitude and phase balance.
- Higher return loss (lower VSWR) at the primary side.

Impedance Matching using Transmission Lines

1. Impedance Matching using Quarter-Wavelength ($\lambda/4$) Transmission Line

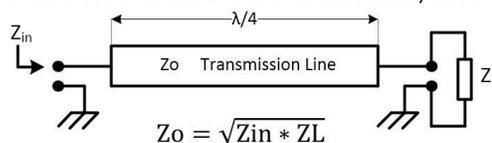
An impedance transformer may be realized by inserting a segment of a different transmission line with appropriate characteristic impedance.

A quarter-wavelength ($\lambda/4$) impedance transformer is a component used in RF engineering, and is consisting of a transmission line with length of $\lambda/4$ and terminated in some known impedance Z_L .

- Although quarter-wavelength transformer can in theory used to match complex impedances, it is more common to use it to match **real impedances**.
- However, a complex load impedance can always be transformed to a real impedance by adding the correct series or shunt reactive component.

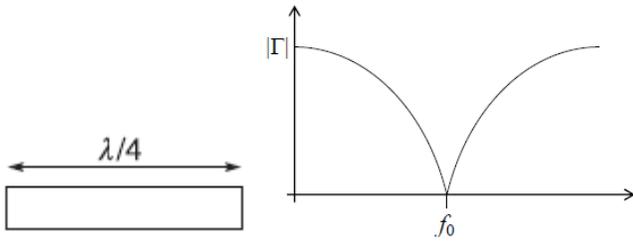
At the operating frequency, the electrical length of the matching section is $\lambda/4$. But at other frequencies the length is different, so a perfect match is no longer obtained.

The quarter-wavelength transformer has a limited bandwidth, like other transformation methods.



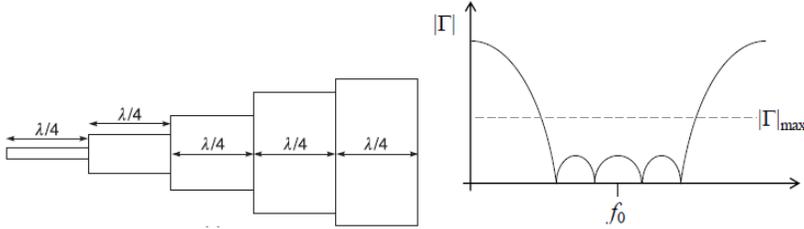
The characteristic impedance Z_o of the quarter-wave line ($\lambda/4$) is the geometric average of Z_{in} and Z_L .

a)



A quarter-wave $\lambda/4$ transformer provides a perfect match at only one frequency.

b)

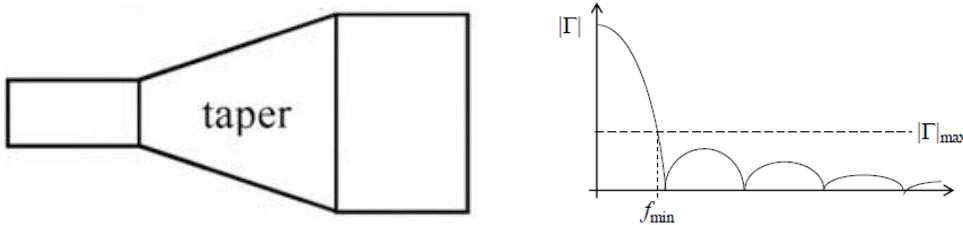


A broadband design may be obtained by a cascade of $\lambda/4$ line segments of gradually varying their characteristic impedance.

It is not possible to obtain exactly zero reflection coefficient for all frequencies in the desired band. Therefore, available design approaches specify a maximum reflection coefficient (or maximum VSWR) which can be tolerated in the frequency band of operation.

- The change of characteristic impedances Z_n must increase or decrease monolithically.

c)



Another broadband matching approach may use a tapered line transformer with continuously varying characteristic impedance along its length (characteristic impedance varies continuously in a smooth fashion).

In this case, the design obtains reflection coefficients lower than a specified tolerance at frequencies exceeding a minimum value.

- The required length of the taper segment should be about 0.5 to 1.5 of wavelength.

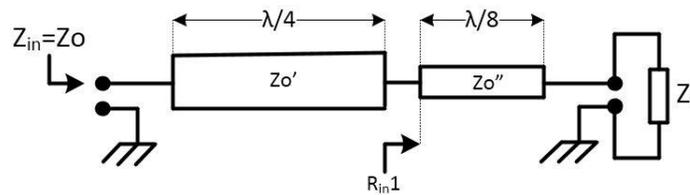
A different narrow-band approach involves the insertion of a shunt imaginary admittance on the line. Often, the admittance is realized with a segment (or stub) of transmission line and the technique is commonly known as stub matching. The end of the stub line is short-circuited or open-circuited, in order to realize an imaginary admittance.

A second narrow-band example involves the insertion of series impedance (stub) along the line.



2. Impedance Matching of Complex Loads using $\lambda/4$ and $\lambda/8$ Transmission Lines

Sometimes, when doing impedance matching of complex loads ($Z_L = R_L + jX_L$), a combination of two-line segments connected in cascade is used, one line with length $\lambda/4$ and the other line with length $\lambda/8$, as is shown in figure below:



Impedance matching using $\lambda/4$ and $\lambda/8$ Transmission Lines

- This matching circuit is based on the fact that, a lossless transmission line with length $\lambda/8$ has a real input impedance when its characteristic impedance Z_0 has the value $|Z_L|$.

Once the real impedance R_{in1} is obtained at the entrance to the line segment $\lambda/8$, the transformation of this resistance R_{in1} into the value necessary for matching, Z_0 , is obtain using the line with length $\lambda/4$. Thus, in the circuit from the figure above, the characteristic impedance of the two-line segments should have the following values:

$$Z_0'' = |Z_L| = \sqrt{R_L^2 + X_L^2}$$

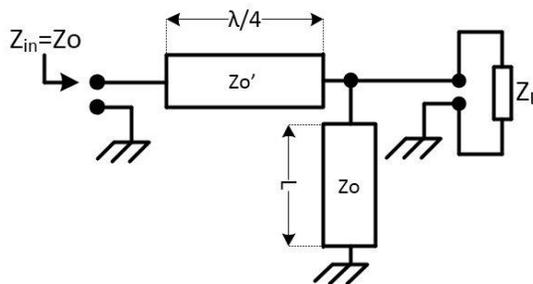
$$Z_0' = \sqrt{Z_0 \cdot R_{in1}}$$

- In principle, this circuit variant can be used to match any complex load.
- Basically, the limitations here are represented by the frequency bandwidth, by the efficiency of the circuit, but first of all by the possibility of achieving the characteristic impedances Z_0' and Z_0'' .

3. Impedance Matching with Reactive Compensation of the Complex Load

In case of a complex load, a line segment with length $\lambda/4$ can be used for matching, if previously the reactive part of the load was compensated.

This compensation can be obtained by connecting in parallel (shunt) with the load a short-terminated line segment having a characteristic impedance Z_0 equal to that of the input impedance, $Z_0=Z_{in}$.



Impedance Matching with reactive compensation of the complex load

The **Admittance Y** equation is: $Y = G + jB = 1/Z$ (where: G=conductance, B=susceptance, Z=impedance)

- The length L of the line segment in parallel with the load is chosen so its input **Susceptance B_{in}** compensate for the load **Susceptance B_L**. An RF simulator can be used to find the value of L .
- The line segment $\lambda/4$ serves to transform the load **Conductance G_L** remaining after compensation, into the resistive value required to match the transmission line:

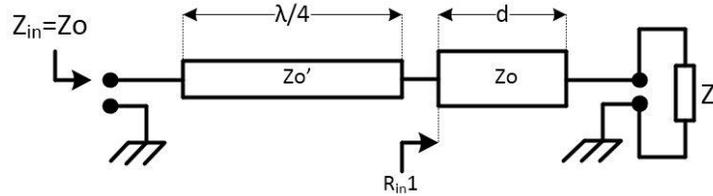
$$Z_0' = \sqrt{Z_0/G_L}$$

- In principle the presented matching circuit could be used for matching any complex load, but practically are limitations of achieving the: necessary impedance of the Z_0' transmission line, the frequency bandwidth, and the efficiency of the circuit.

4. Impedance Matching of Complex Loads using an interspersed $\lambda/4$ line segment

The following circuit shows another possibility to do impedance matching for complex loads.

In this configuration the role of the line segment with length d is to transform the complex value of the load Z_L into a resistive value R_{in1} at its input. The resistance thus obtained will be transformed into the necessary resistance Z_0 with the help of the line segment with length $\lambda/4$.



Impedance Matching using an interspersed line segment

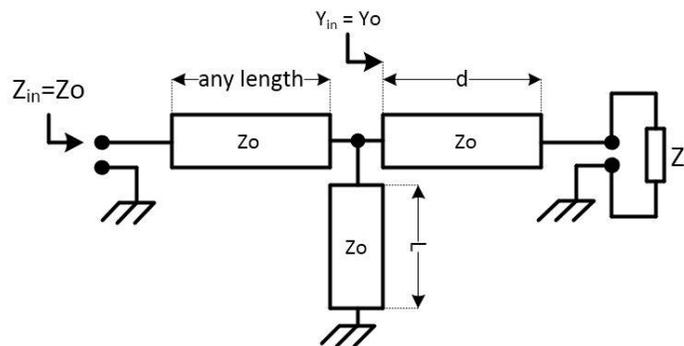
- The determination of the length d can be performed analytically or graphically on the Smith chart diagram. An RF or an EM CAD simulator would provide also the length of the d segment.
- Analytically, the distance d results from the condition that the imaginary part of the input impedance in the line segment terminated on the load Z_L is canceled. Once the resistance R_{in1} has been determined, also can be calculated the characteristic impedance Z_0' of the $\lambda/4$ line segment:

$$Z_0' = \sqrt{(Z_0 * R_{in1})}$$

- The analysis admits two solutions with two distinct values of Z_0' , one bigger than Z_0 and other smaller than Z_0 .
- The circuit have practical limitations of achieving: the necessary impedance of the transmission line Z_0' , the frequency bandwidth, and the efficiency of the circuit.

5. Impedance Matching of Complex Loads using a parallel (shunt) line segment

Any complex load Z_L can be matched to an access line using only transmission lines with the same characteristic impedance $Z_0 = Z_{in}$. This type of matching network consists of a short-circuited line segment with length L , connected in parallel (shunt) on the line, at a certain distance d from the load, as in the figure below:



Impedance matching using a line segment connected in parallel (shunt)

- Realization of this matching circuit is conditioned by the choice of the lengths d and L .
- Due to the parallel connection, the analysis with the Admittance Y is preferred.
- The lengths d and L , which are required to match a given load Z_L , can be determined either analytically, or determined graphically using the Smith chart diagram. An RF or an EM CAD simulator would provide also the solution finding the d and L lengths.
- This matching circuit has limited bandwidth and is accurate only at the design frequency, the frequency at which the lengths d and L were defined.

Impedance matching using L-C components

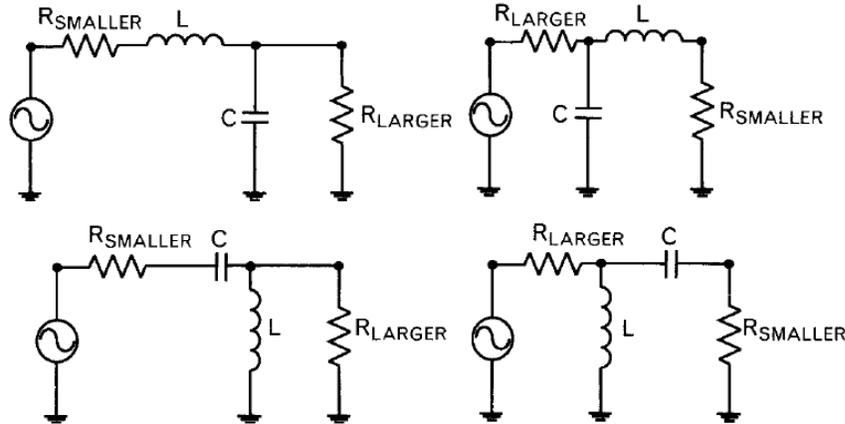
Any two resistive terminations can be simultaneously matched by adding two reactive elements between them.

If we need to match in a narrow frequency a source R_S and a load R_L , we can get almost the same performance by using a high-pass or low-pass network configuration.

The pass-band performances near the matching frequency are very similar for both networks, when the out-of-band characteristics of the low-pass and high-pass are different.

A low-pass rejects signals at the high-end, and allow passing at low frequencies.

The high-pass network does the opposite.



Four possible single matching L-C networks

To increase the impedance level, the series matching element must be placed next to the termination with smaller resistance.

Steps to follow for Impedance Matching using two reactive elements:

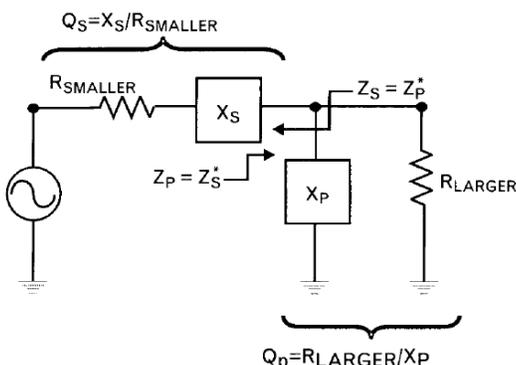
1) Add a series reactive element next to $R_{SMALLER}$ and a parallel one to R_{LARGER} . Series element could be either Inductor or Capacitor but the parallel one must be opposite type.

If the series element is an inductor, we create a low-pass topology, and when the series element is a capacitor, we get a high-pass topology.

- In case of transistor matching, usually the gain of the transistor is higher at lower frequencies, so there may be a low-frequency stability problem. In such a case, sometimes a high-pass LC network at the input (series C, shunt L) may be more stable.
- At the output of the transistor harmonic filtering is required this can be done with a low-pass matching network (series L, shunt C).

2) Add a series reactive element to $R_{SMALLER}$ and a parallel one to R_{LARGER} , forms two sub-networks, one inductive and other capacitive (one in series and one in parallel), and they must represent complex conjugates impedances to each other, at design frequency.

Therefore, the Q-factors of these two sub-networks must be equal at the matching frequency.



$$Q = Q_S = Q_P = \sqrt{\frac{R_{LARGER}}{R_{SMALLER}} - 1}$$

3) Knowing the Q-values, we can find the series and parallel elements, reactances, and get the values of the inductor and capacitor of the network using the following equations:

$$L_{nH} = \frac{0.159 X_L}{f_{GHz}} = \frac{0.159 Q_S R_{SMALLER}}{f_{GHz}} = \frac{0.159 R_{LARGER}}{f_{GHz} Q_P}$$

$$C_{pF} = \frac{159}{f_{GHz} X_C} = \frac{159}{f_{GHz} Q_S R_{SMALLER}} = \frac{159 Q_P}{f_{GHz} R_{LARGER}}$$

Example:

To match a 5 Ω to a 50 Ω resistive load at 850MHz, we can add a series inductor to R_{SMALLER} (5 Ω) and a shunt capacitor to R_{LARGER} (50 Ω).

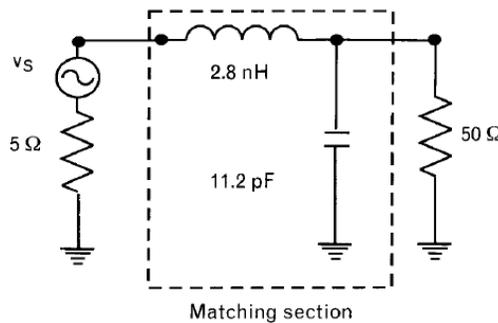
To calculate the required Q-factors for the new sub-networks:

$$Q_S = Q_P = \sqrt{\frac{R_{LARGER}}{R_{SMALLER}} - 1} = \sqrt{\frac{50}{5} - 1} = 3$$

To find the inductor and the capacitor values at 850MHz we use:

$$L_S = \frac{0.159 Q_S R_{SMALLER}}{f_{GHz}} = \frac{0.159(3)5}{0.85} = 2.8 \text{ nH}$$

$$C_P = \frac{159 Q_P}{f_{GHz} R_{LARGER}} = \frac{159(3)}{0.85(50)} = 11.2 \text{ pF}$$



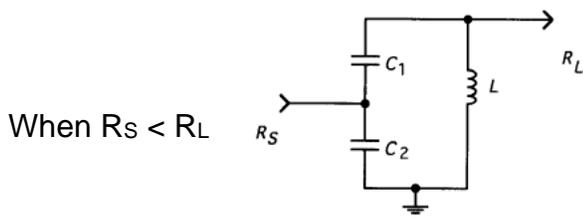
- A low-pass L-C network used to match two resistive terminations shows an asymmetric broadband frequency response, with negligible loss near the matching frequency.
- Loss below matching frequency is caused by the mismatch between the two terminations.

$$\text{Mismatch Loss}_{[dB]} = 10 \times \text{LOG} (1 - \Gamma^2)$$

- Loss above matching frequency is caused by the roll-off the low-pass network (12dB/octave slope).
- An L-C matching network response shows symmetry only near the matching frequency, and the 3dB bandwidth is meaningless if the mismatch loss between the two terminations is less than 3dB.

Tapped Capacitive Impedance Transformer

The tapped capacitor circuit is another approximate method for obtaining impedance level transformation.



$$R_L \approx R_S \left(\frac{C_1 + C_2}{C_1} \right)^2 \quad C_2 = C_1 \left(\sqrt{\frac{R_L}{R_S}} - 1 \right)$$

The maximum value of the inductance L_{MAX} used by the tapped impedance transformer is:

$$L_{max} = \frac{R_L}{\omega \sqrt{\frac{R_L}{R_S} - 1}}$$

The upper limit of L is when C_2 is not required, while at low L/C ratios the circuit becomes sensitive to small component value changes.

As a general rule, L must be smaller than L_{MAX} but not very much smaller.

Example:

If $C_1 = 100\text{pF}$, and $C_2 = 200\text{pF}$, we can match: $R_S = 50 \Omega$ with $R_L = 450 \Omega$
 If the matching frequency is 100MHz , the maximum value of parallel inductance will be, $L = 253\text{nH}$

Bandwidth of the Matching Networks

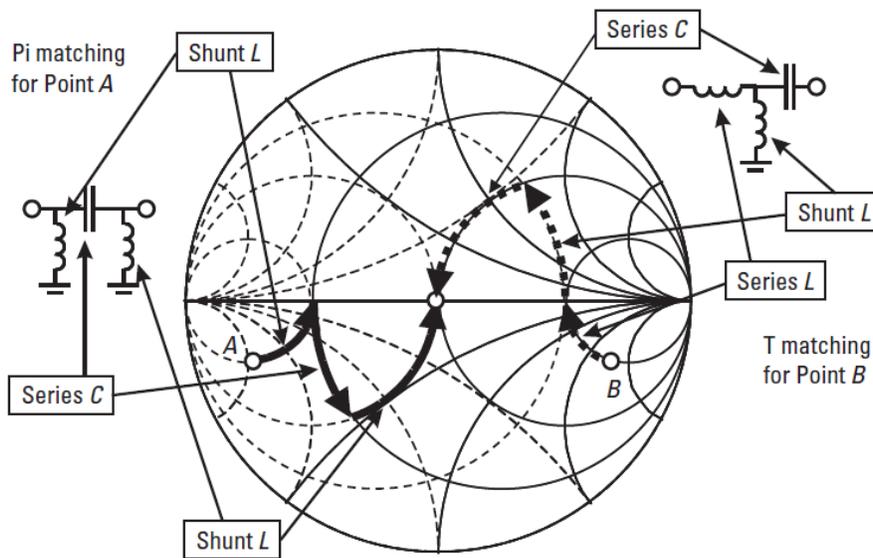
In the example above matching network can be either low-pass or high-pass. Comparing the frequency responses of the low-pass and high-pass networks shows considerable difference.

- The low-pass network absorbs the parasitics on both sides, while the high-pass circuit resonates them, resulting in narrower bandwidth.

If the bandwidth obtained by a single section L-C network is not sufficient, we can increase it by adding another section. Instead transforming the impedance directly from one to another, first transform to an intermediate impedance R_{INT} and finish the matching circuit with a second L-C section.

Advantage of the **Tee** or **Pi** networks is: using an extra element there is an extra degree of freedom to control the value of Q_n in addition to performing impedance transformation/matching.

- For equal component sensitivities set the $R_{INT} = \sqrt{R_S \times R_L}$
- R_{INT} is not an actual component: it only indicates the intermediate impedance level.
- If the increase is not sufficient, adding more sections brings further bandwidth improvement. The bandwidth improvement is caused by reducing the termination ratio, which in turn reduces the Q of the matching sections.
- When 3 elements are used in a matching network, we are no longer limited to a single value of network Q as using 2 element circuit. For a given set of source/load resistances now we can select any Q higher than the one when using 2 element L type network.
- The 3 element **Pi** or **Tee** type networks can match a source that is either higher or lower than the load resistance, simply by alternating ratio of the two shunt components.



Pi and T (tee) type matching networks on Smith Chart

The **Pi network** can be described as two “back-to-back” L networks that are both configured to match the load and the source to a virtual resistance located at the junction between the two networks. The virtual resistance R_{INT} must be smaller than either R_s or R_L because it is connected to the series arm of each L section.

R_{INT} is defined by the desired Loaded-Q of the circuit that was specified by the design process.

$$Pi \text{ Loaded} - Q = \sqrt{\left(\frac{R_{LARGEST}}{R_{INT}}\right)} - 1$$

where $R_{LARGEST}$ is the largest terminating resistance R_s or R_L .

The **Tee network** design is the same as for Pi network except that with Tee you match the load and the source, through two L-type networks to a virtual R_{INT} resistance which is larger than either R_s or R_L . The Tee network is often used to match two low valued impedances when a high-Q arrangement is needed. The Loaded-Q of the Tee network is determined by the L section that has the highest Q. The L section with highest Q will occur at the end which has the smallest terminating resistance (each terminating resistor is in the series leg of each network).

$$Tee \text{ Loded} - Q = \sqrt{\left(\frac{R_{INT}}{R_{SMALLEST}}\right)} - 1$$

where $R_{SMALLEST}$ is the smallest terminating resistance R_s or R_L .

Impedance Matching of Complex Terminations

- In case of complex loads maximum power will be transferred when source is conjugately matched to a load. So, transform one complex termination to the complex conjugate of the second termination.
- If a load is purely resistive, the source should appear to be resistive with the same value.
- If the load is complex the source should appear to have the same resistance but the opposite reactance. The two reactances will then cancel (resonate) leaving only the identical source and load resistances.
- When a network transforms a given load to complex conjugate of a given source, the reverse is also true. The source is simultaneously transformed by the same network into complex conjugate of the load.

When one or both of the impedances to be matched already has imaginary parts, two possibilities exist for computing (absorption or resonance).

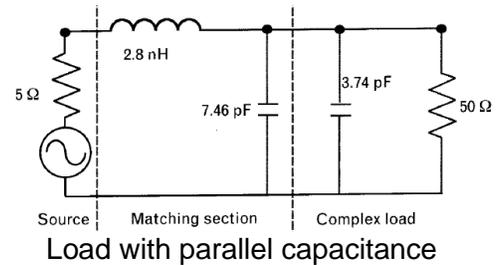
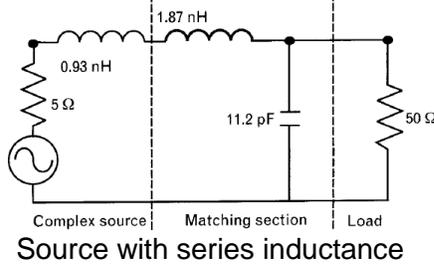
- Absorb the parasitic into matching network.
- Resonating the excessive parasitic inductance or parasitic capacitance.

- Source and load inductance or capacitance may be integrated into the matching network as long as the Q-factor of the termination does not exceed the computed nodal-Q of the L-C network. The limit is set by the resistance ratio of the terminations to be matched.

When the parasitic of termination exceed the maximum value that the matching network can absorb use one of the two situations:

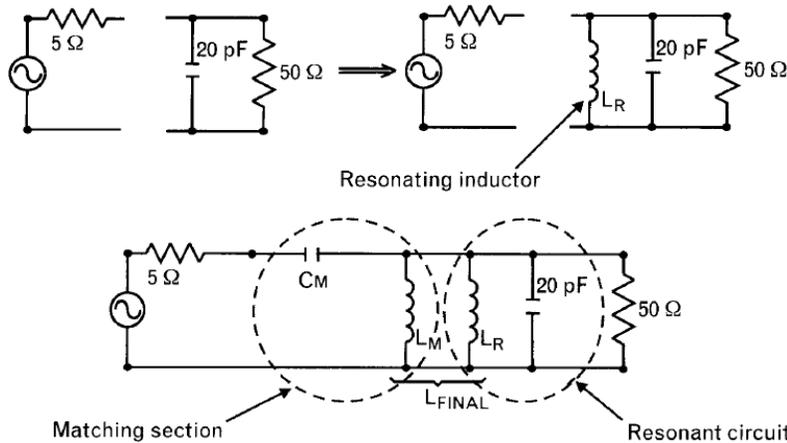
- Fully resonate the parasitics and proceed matching to the leftover resistive part of the termination
- Resonate only the excessive part of the parasitic and use the remainder as part of the matching circuit.

- Both these resonance-based techniques reduce the bandwidth, the more reactance we have to resonate, the narrower the frequency response becomes.



When the Q of a termination exceeds the limit set by the nodal-Q calculations, we need to neutralize the excessive amount of parasitic reactance.

In the example below, the 20pF parallel load capacitance is 8.8pF higher than the maximum limit, set by the computed Q of 3.0, and it is fully resonated by the inductor L_R .



$$L_R = \frac{25.33}{f_{RGHz}^2 C_{pF}} = \frac{25.33}{0.85^2 (20)} = 1.75 \text{ nH}$$

$$L_M = \frac{0.159 R_{LARGER}}{f_{GHz} Q_P} = \frac{0.159(50)}{0.85(3)} = 3.12 \text{ nH}$$

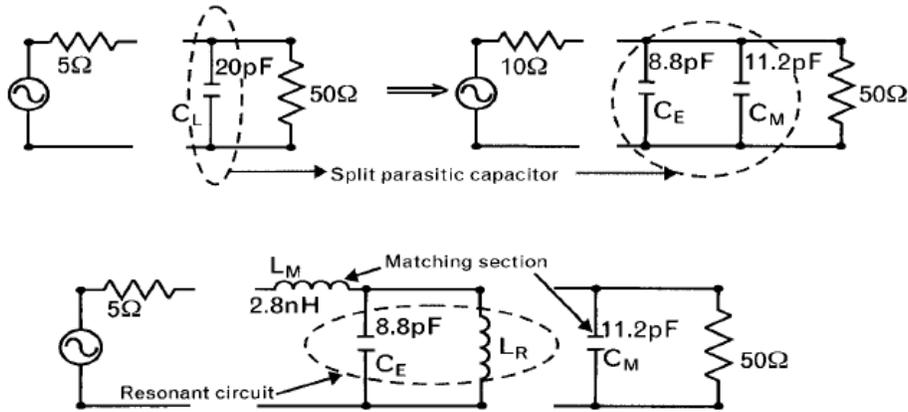
$$C_M = \frac{159}{f_{GHz} Q_S R_{SMALLER}} = \frac{159}{0.85(3)5} = 12.47 \text{ pF}$$

after that we can combine L_R and L_M in a single parallel inductor L_{MR}

$$L_{MR} = \frac{L_R (L_M)}{L_R + L_M} = \frac{1.75(3.12)}{1.75 + 3.12} = 1.12 \text{ nH}$$

If the inductor L_R is selected to resonate 8.8pF of the 20pF load capacitance, the remaining 11.2pF can be used with inductor L_M to match the two terminations.

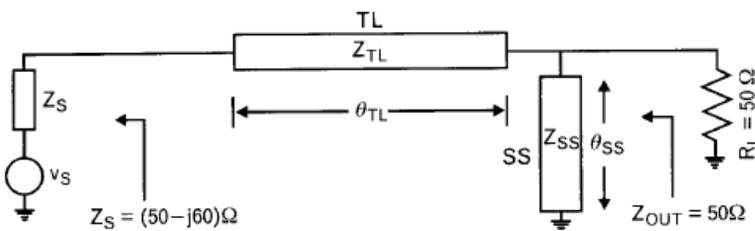
We get more bandwidth this approach compared to the circuit shown above.



Impedance Matching with Transmission Lines using Smith Chart

Smith Chart is a good choice when Impedance Matching is done using transmission lines.

- Cascading transmission lines always follow a clockwise rotation on the Smith Chart.
- Moving away from a termination on a transmission line, always follow a clockwise circular rotation on the Smith Chart.
- If the chart is normalized to the characteristic impedance of the transmission line, the rotation is along a concentric circle.
- The radius of the concentric circle is determined by the normalized termination.

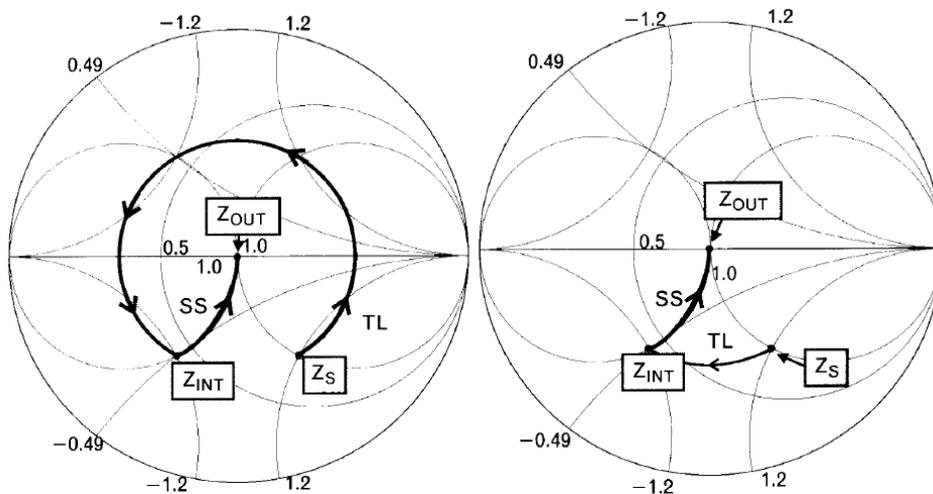


A complex source can be matched to the 50Ω load with a cascade series transmission line and a parallel short-circuited stub.

There are 4 adjustable parameters:
 Z_{TL} , θ_{TL} , Z_{SS} , θ_{SS}

A parallel stub is treated as an equivalent parallel inductor or capacitor at specific frequencies, depending on what type of reactance it represents.

If we use several cascade lines with different characteristic impedances, the Smith chart must always be renormalized to the appropriate impedance.



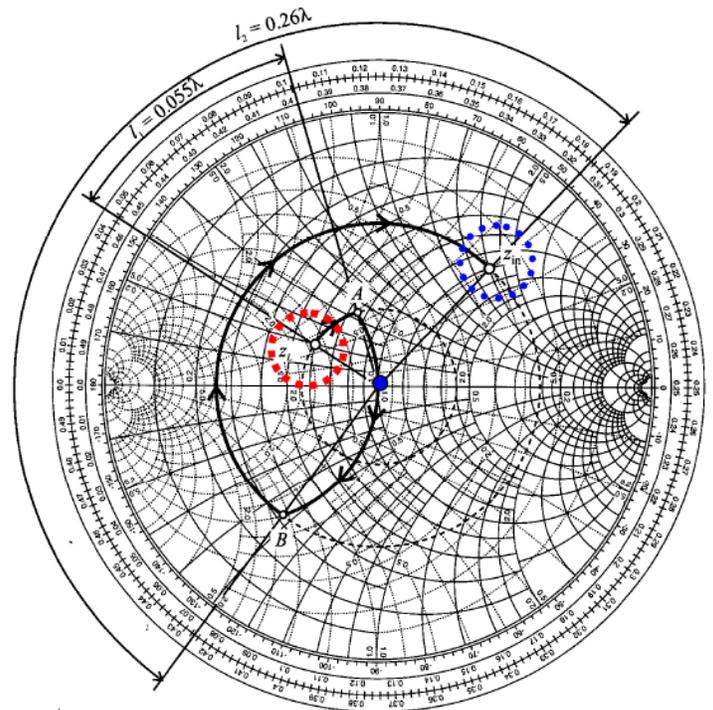
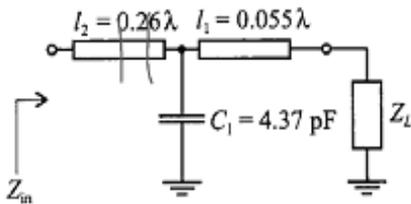
- Following a counter-clockwise rotation on the chart is equivalent to de-embedding, which is incorrect for this application.
- Moving away from any termination (source or load) with a transmission line, always leads to a clockwise rotation.
- In general, one shunt capacitor and two series transmission lines is sufficiently to transform any load to any input impedance.

Example:

Transform a load $Z_L = 30 + j10$, to an input $Z_{in} = 60 + j80$, at frequency $f = 1.5\text{GHz}$

Steps for Impedance Matching with Transmission Lines using Smith chart:

- From Z_L go to point A using a series-TL ($l_1 = 0.055\lambda$)
- From point A go to point B using a shunt capacitor ($C_1 = 4.37\text{pF}$)
- From point B go to Z_{in} using a series-TL ($l_2 = 0.26\lambda$)



Impedance Matching Guide using Smith Chart

A Smith chart is a circular plot with a multitude of interlaced circles on it. Theoretically, using Smith chart, matching impedances with apparently complicated structures, can be done without any computation. The only effort required is the reading and following of values along the circles.

Impedance Matching issues can be analyzed as **trajectories on the Smith Chart**, where the addition of a series or shunt component moves the total impedance along constant impedance Z , admittance Y , or resistance R circles.

The Smith chart is a polar plot of the complex reflection coefficient named **gamma** and symbolized by Γ .

- The reflection coefficient Γ is defined as the ratio between the reflected voltage wave and the incident voltage wave. Because the impedances are complex numbers, the reflection coefficient will be a complex number as well.

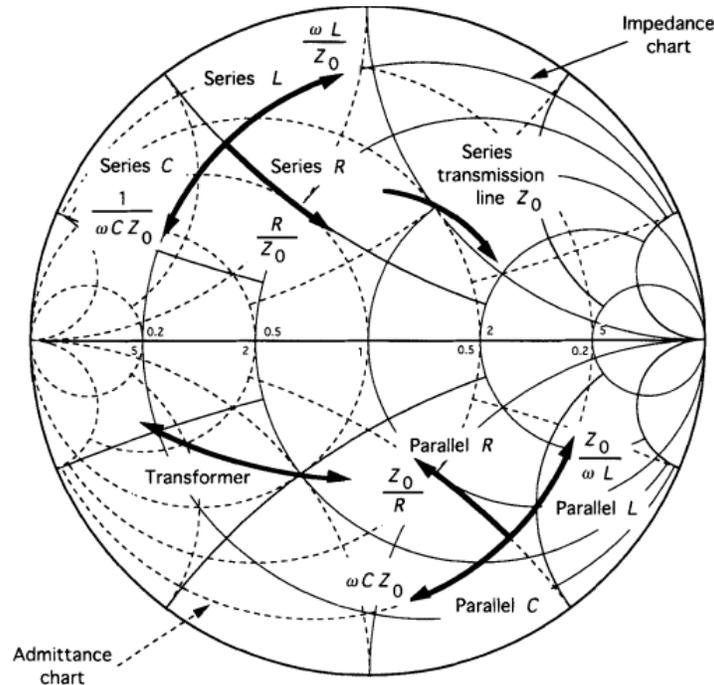
Also, the reflection coefficient could be defined mathematically as the 1-port scattering parameter **s11**.

A Smith chart is developed by examining the load where the impedance must be matched.

- Instead of considering the load impedance directly, should express its reflection coefficient Γ_L , which is used to characterize a load.

The Smith chart is built by considering impedance (resistance and reactance). Once the Smith chart is built, it can be used to analyze these parameters in both the series and parallel (shunt) worlds.

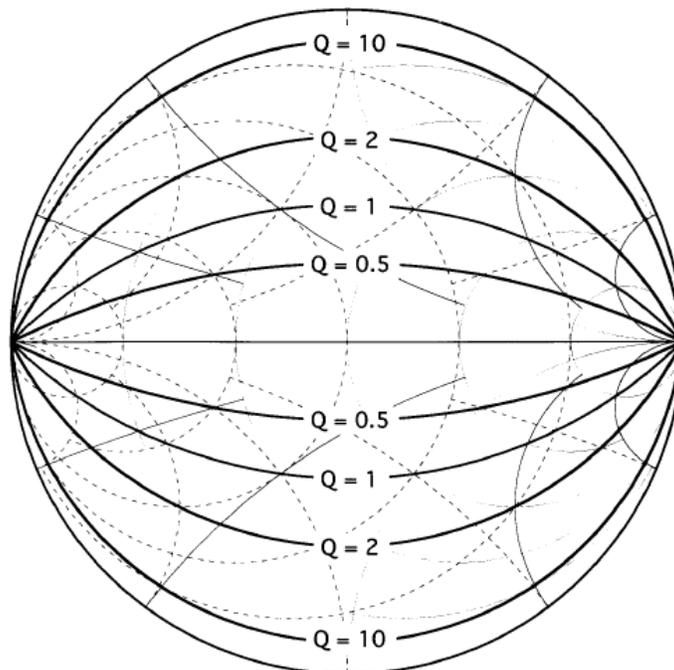
- Adding elements in a series is straightforward. New elements can be added and their effects determined by simply moving along the circle to their respective values.
- Summing elements in parallel (shunt) requires considering additional parameters. Often it is easier to work with parallel elements in the admittance chart.



Trajectories on the Smith Chart function of types of added components on the circuit

- If the task is to match specific impedance to a reference impedance (generally to 50Ω), then the target of the impedance matching is to arrive at the center of the Smith Chart by moving along the arcs from the initial point.
- If the task is to provide impedance matching to an impedance other than the reference impedance, then the end point of the matching trajectory must be the conjugate of the target impedance.

Constant-Q lines can be plotted on the Smith Chart to estimate the matching network bandwidth.



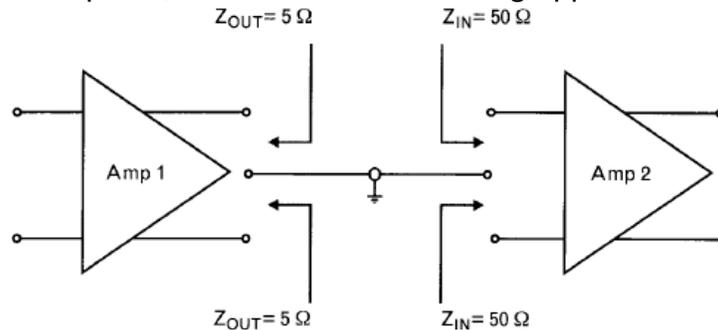
- The closer an Impedance Matching trajectory comes to the edge of the Smith Chart, the narrower the bandwidth.
- Maximum bandwidth for a given matching network can be obtained by keeping the trajectories short, well away from the edges of the Smith Chart, and as close as possible to the real axis.
- If the target is to design a circuit with a specific bandwidth (equal to a certain Q), one vertex of the matching trajectory must touch the desired constant-Q arc, and all other trajectory points should be well inside of lower Q regions.

Impedance Matching of Balanced Circuits

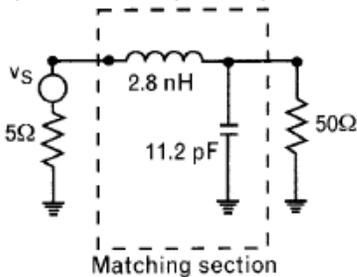
- Introducing a virtual ground between the terminals of the balanced circuit ports, the Impedance Matching of balanced circuits can be reduced to the situation of single-ended circuits.
- Then we can match the individual halves of the balanced circuits the same as as in single-ended circuits.
- After the networks are derived, the virtual ground may be eliminated to save components.

Example:

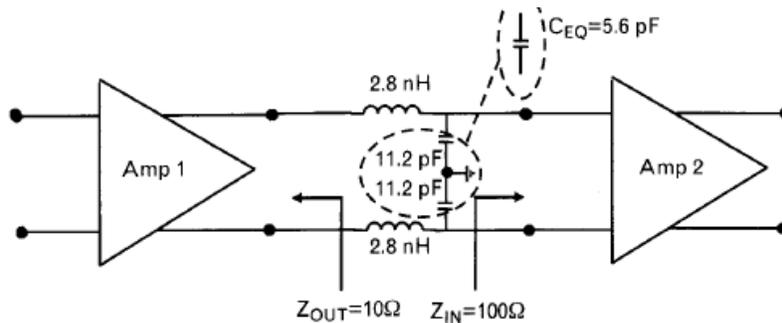
If we have to match at 850MHz the $10\ \Omega$ differential output Z_{OUT} of an amplifier, to the $100\ \Omega$ input impedance Z_{IN} of the second amplifier, we can use the following approach.



With the help of a virtual ground the balanced ports can be split into unbalanced ports. Input and output impedances of the unbalanced ports are half of the balanced ports.

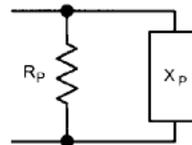
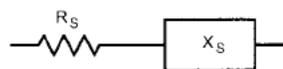
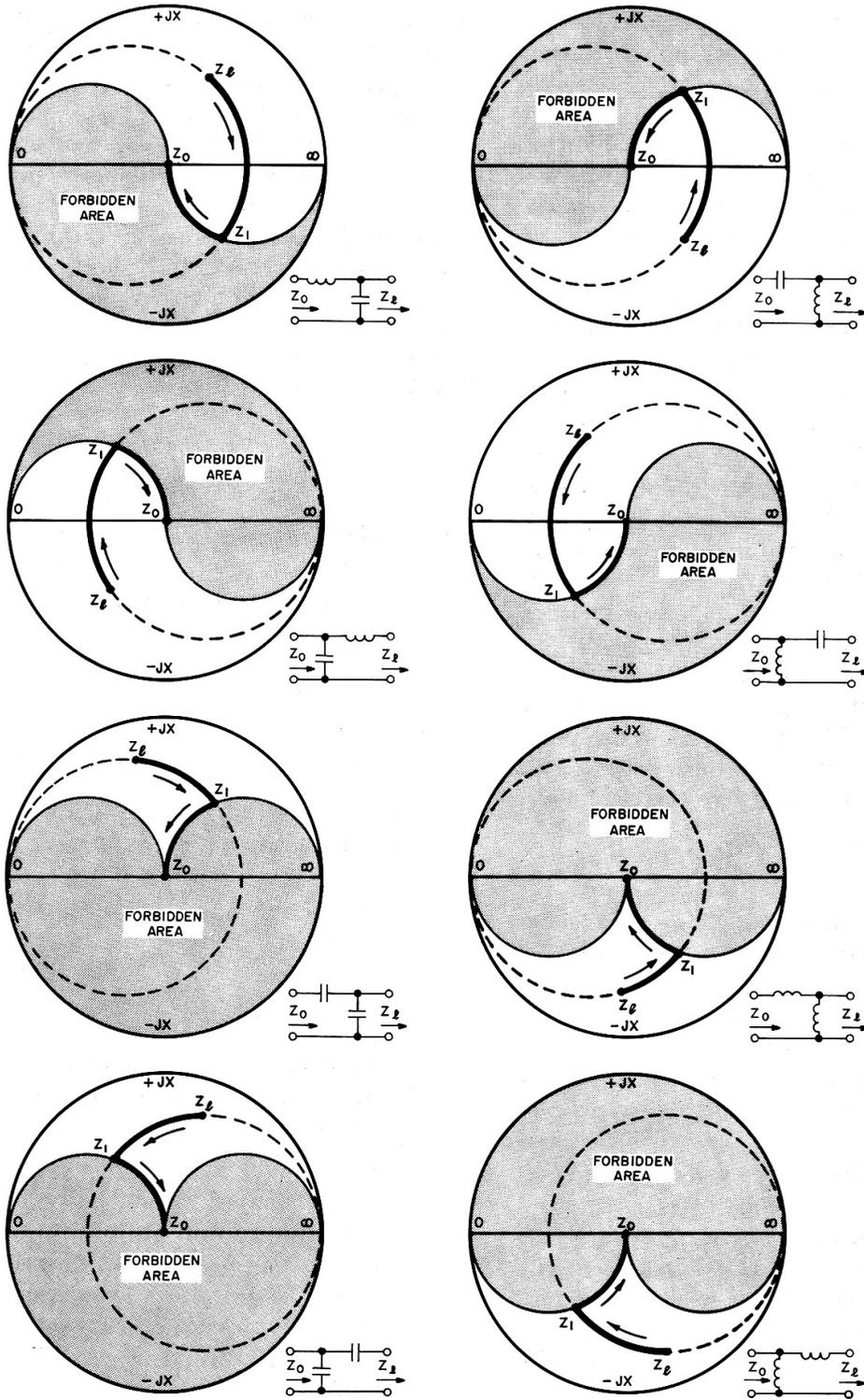


The unbalanced L-C circuit used to match $5\ \Omega$ to $50\ \Omega$ is a standard L-type matching network using a series L and a shunt C.



- After establishing the unbalanced matching circuits, the virtual ground can be eliminated. The two 11.2pF capacitors can be combined into a single floating 5.6pF capacitor. The two balanced ports are now matched to each other at desired frequency.

Annex 1: Impedance Matching Networks - Smith Chart Regions, Configurations and Equations

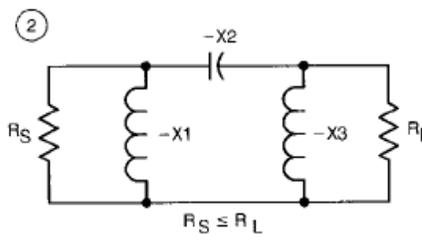
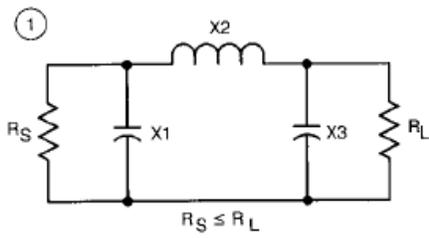


$$R_S = R_P \cdot \left[\frac{X_P^2}{R_P^2 + X_P^2} \right];$$

$$X_S = X_P \cdot \left[\frac{R_P^2}{R_P^2 + X_P^2} \right]$$

$$R_P = R_S \cdot \left[\frac{R_S^2 + X_S^2}{R_S^2} \right];$$

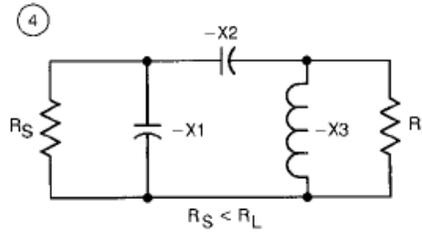
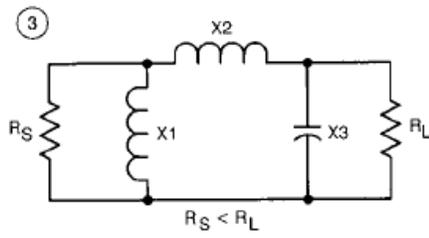
$$X_P = X_S \cdot \left[\frac{R_S^2 + X_S^2}{X_S^2} \right]$$



$$X1 = -R_S \sqrt{\frac{R_L/R_S}{Q^2 + 1} - (R_L/R_S)}$$

$$X2 = \frac{Q \times R_L - (R_S \times R_L / X1)}{Q^2 + 1}$$

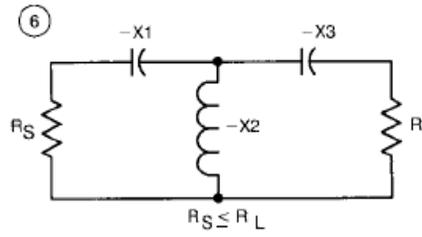
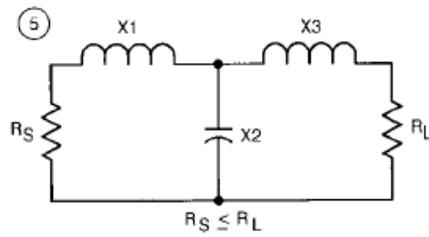
$$X3 = -\frac{R_L}{Q}$$



$$X1 = \frac{R_S}{\sqrt{\frac{R_S(Q^2 + 1)}{R_L} - 1}}$$

$$X2 = \frac{R_L \times Q}{Q^2 + 1} \left(1 - \frac{R_S}{Q \times X1}\right)$$

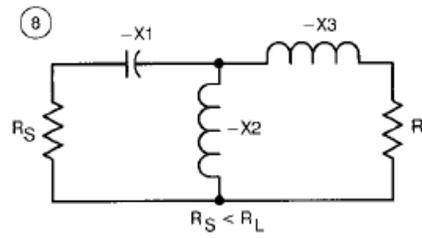
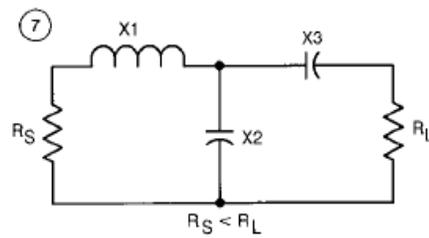
$$X3 = -\frac{R_L}{Q}$$



$$X1 = R_S \times Q$$

$$X2 = \frac{-R_S(1 + Q^2)}{Q + \sqrt{\frac{R_S(1 + Q^2)}{R_L} - 1}}$$

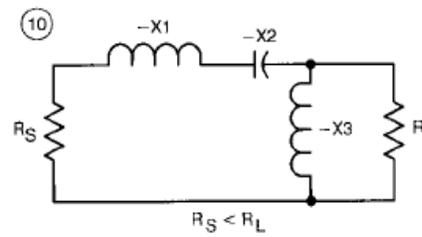
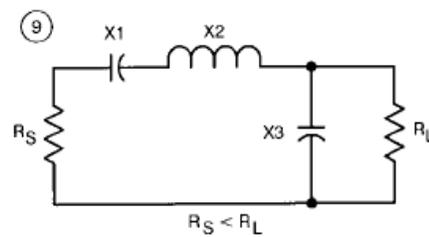
$$X3 = R_L \sqrt{\frac{R_S(1 + Q^2)}{R_L} - 1}$$



$$X1 = Q \times R_S$$

$$X2 = \frac{-R_S(1 + Q^2)}{Q - \sqrt{\frac{R_S(1 + Q^2)}{R_L} - 1}}$$

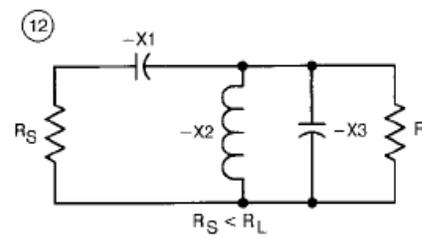
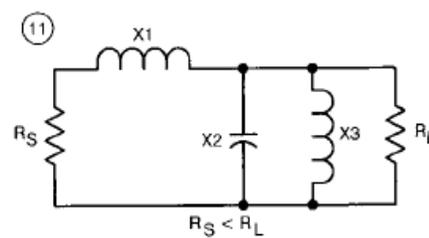
$$X3 = -R_L \sqrt{\frac{R_S(1 + Q^2)}{R_L} - 1}$$



$$X1 = -Q \times R_S$$

$$X2 = \sqrt{R_S \times R_L - R_S^2} - X1$$

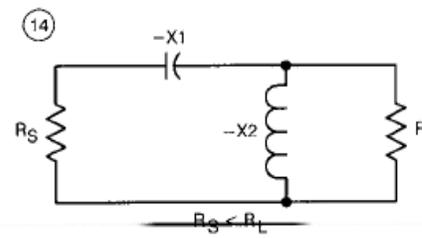
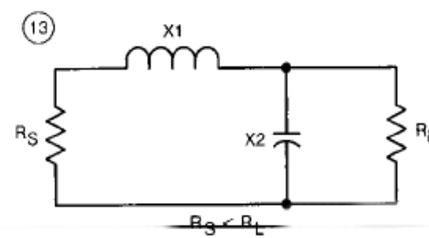
$$X3 = \frac{-R_S \times R_L}{X1 + X2}$$



$$X1 = R_S \sqrt{\frac{R_L}{R_S} - 1}$$

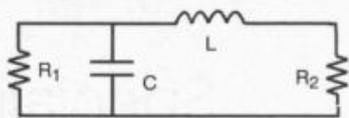
$$X3 = \frac{R_L}{Q}$$

$$X2 = \frac{-X3}{\frac{X1}{Q \times R_S} + 1}$$

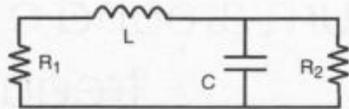


$$X1 = \sqrt{R_S \times R_L - R_S^2}$$

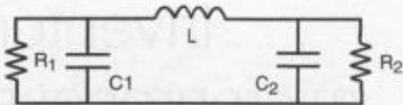
$$X2 = -\frac{R_S \times R_L}{X1}$$



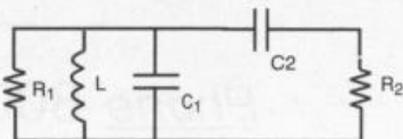
1a: L Section (a)



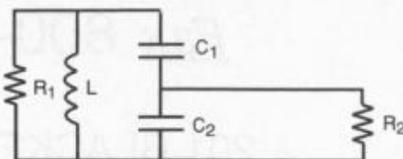
1b: L Section (b)



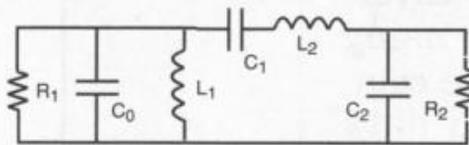
2: Pi Network



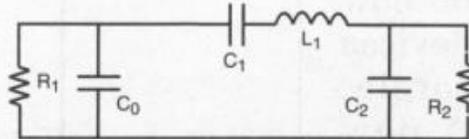
3: Gamma Network



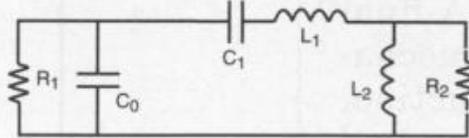
4: Tapped Capacitor



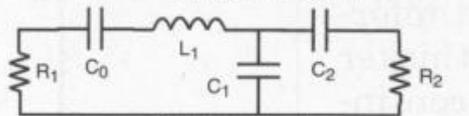
5: Modified PI Network I



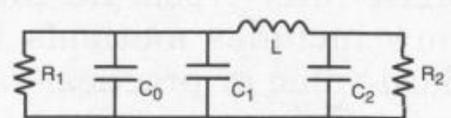
6: Modified PI Network II



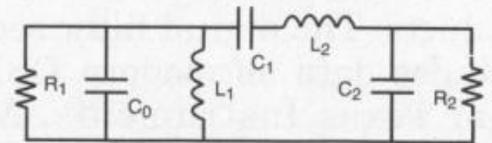
7: Modified PI Network III



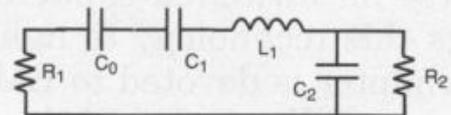
8: Modified T Network



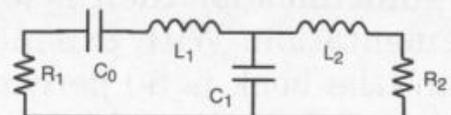
9: PI Network With Input C0



10: Modified PI Network IV



11: Modified L Network



12: T Network

1a: L section, Capacitive Input

Restriction: $R_1 > R_2$

$$X_L = \sqrt{R_1 R_2 - R_2^2}$$

$$X_C = \frac{R_1 R_2}{X_L}$$

$$Q = \frac{X_L}{R_2} = \frac{R_1}{X_C}$$

1b: L section, Inductive Input

Restriction: $R_1 < R_2$

$$X_C = R_1 \sqrt{\frac{R_1}{R_2 - R_1}}$$

$$X_L = \frac{R_1 R_2}{X_C}$$

$$Q = \frac{X_L}{R_1} = \frac{R_2}{X_C}$$

2: Pi network

Restrictions: $R_1 > R_2$, $Q > \sqrt{\frac{R_1}{R_2} - 1}$

$$X_{C1} = \frac{R_1}{Q}$$

$$X_{C2} = R_2 \sqrt{\frac{R_1 / R_2}{Q^2 + 1 - (R_1 / R_2)}}$$

$$X_L = \frac{QR_1 + R_1 R_2 / X_{C2}}{Q^2 + 1}$$

3: Gamma network

Restrictions: $R_1 > R_2$, $Q > \frac{X_{C2}}{R_2}$

$$X_L = \frac{R_1}{Q}$$

$$X_{C2} = R_2 \sqrt{\frac{R_1}{R_2} - 1}$$

$$X_{C1} = \frac{R_1}{Q} - \frac{1}{\left(1 - \frac{X_{C2}}{QR_2}\right)}$$

4: Tapped Capacitor

Restrictions: $R_1 > R_2$, $Q > \sqrt{\frac{R_1}{R_2} - 1}$

$$X_L = \frac{R_1}{Q}$$

$$X_{C_1} = \frac{R_1 Q}{Q^2 + 1} \left(1 - \frac{R_2}{Q X_{C_2}} \right)$$

$$X_{C_2} = \frac{R_2}{\sqrt{(Q^2 + 1) \frac{R_2}{R_1} - 1}}$$

5: Modified Pi network

Restrictions: $R_2 > R_1$, $\frac{R_2}{R_1} > \frac{Q^2}{Q^2 + 1}$

$$X_{C_1} = QR_1$$

$$X_{C_2} = \frac{R_2}{\sqrt{\frac{R_2(Q^2 + 1)}{R_1 Q^2} - 1}}$$

$$X_{L_1} = \frac{QR_1}{\left(\frac{QR_1}{X_{C_0}} \right) + 1}$$

$$X_{L_2} = QR_1 \left(1 + \frac{R_2}{Q X_{C_2}} \right)$$

6: Modified Pi network II

Restrictions:

$$Q > \frac{R_1}{X_{C_0}}; R_1 R_2 > X_{C_0}^2 \frac{Q^2}{Q^2 + 1}$$

$$X_{C_1} = Q \frac{X_{C_0}^2}{R_1} \left(1 - \frac{R_1}{Q X_{C_0}} \right)$$

$$X_{C_2} = \frac{R_2}{\sqrt{\frac{Q^2 + 1}{Q^2} \frac{R_1 R_2}{X_{C_0}^2} - 1}}$$

$$X_{L_1} = Q \frac{X_{C_0}^2}{R_1} \left(1 + \frac{R_2}{Q X_{C_2}} \right)$$

7: Modified Pi network III

Restrictions:

$$Q > \frac{\sqrt{R_1 R_2}}{X_{C_0}}; Q > \frac{R_1}{X_{C_0}}$$

$$X_{L_1} = Q \frac{X_{C_0}^2}{R_1} \left(1 - \frac{\sqrt{R_1 R_2}}{Q X_{C_0}} \right)$$

$$X_{L_2} = X_{C_0} \sqrt{\frac{R_2}{R_1}}$$

$$X_{C_1} = Q \frac{X_{C_0}^2}{R_1} \left(1 - \frac{R_1}{Q X_{C_0}} \right)$$

$$X_{C_2} = \frac{R_2}{Q} \left(\frac{Q X_{C_0}}{\sqrt{R_1 R_2}} - 1 \right)$$

8: Modified T network

Restrictions:

$$R_2 > R_1; Q > \sqrt{\frac{R_1}{R_2} - 1}$$

$$X_{L_1} = QR_1 + X_{C_0}$$

$$X_{C_2} = \sqrt{\frac{R_1}{R_2} (1 + Q^2) - 1}$$

$$X_{C_1} = R_1 \frac{1 + Q^2}{Q - \sqrt{\frac{R_1}{R_2} (1 + Q^2)}}$$

9: Pi network w/ input C_0

Restrictions:

$$R_1 > R_2; Q > \sqrt{\frac{R_1}{R_2} - 1}$$

$$X_{C_0} + X_{C_1} = \frac{R_1}{Q}$$

$$X_{C_2} = R_2 \sqrt{\frac{R_1 / R_2}{(Q^2 + 1) - (R_1 / R_2)}}$$

$$X_L = \frac{QR_1 + \frac{R_1 R_2}{X_{C_2}}}{Q^2 + 1}$$

10: Modified Pi network IVRestrictions: $R_2 > R_1$

$$X_{L1} = X_{C0}$$

$$X_{C1} = QR_1$$

$$X_{C2} = R_2 \sqrt{\frac{R_1}{R_2 - R_1}}$$

$$X_{L2} = X_{C1} + \frac{R_1 R_2}{X_{C2}}$$

11: Modified L networkRestrictions: $R_2 > R_1$

$$X_{C1} = QR_1$$

$$X_{C2} = R_2 \sqrt{\frac{R_1}{R_2 - R_1}}$$

$$X_{L2} = X_{C1} + \frac{R_1 R_2}{X_{C2}} + X_{C0}$$

12: T network

Restrictions:

$$R_2 = R_1; Q > \sqrt{\frac{R_2}{R_1} - 1}$$

$$X_{L1} = R_1 Q + X_{C0}$$

$$X_{L2} = R_2 \sqrt{\frac{R_1}{R_2} (Q^2 + 1) - 1}$$

$$X_{C1} = \frac{R_1 (1 + Q^2)}{Q + \sqrt{\frac{R_1}{R_2} (1 + Q^2) - 1}}$$

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