

Notch Filter

In the September/October 2006 *Morseman* column, I asked readers to identify the circuit in figure 1 to be identified, and to explain its operation. Nobody submitted an answer!

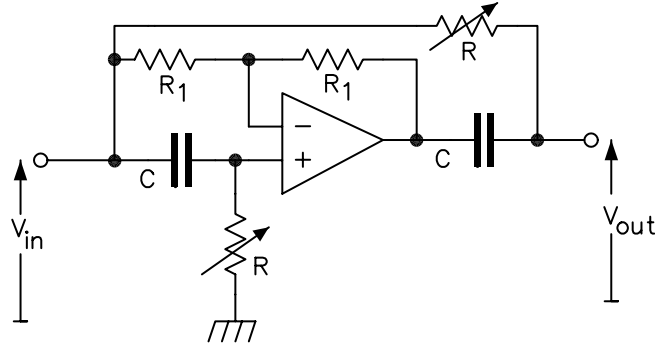


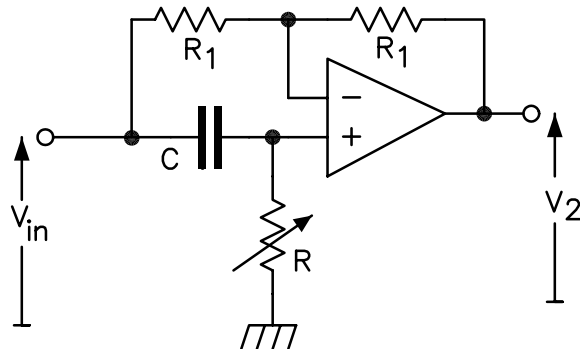
Figure 1: Unknown circuit for pondering

The explanation was given in the *Morseman* column of November/December 2006, but figure 3 on page 19 was incorrectly formatted, making it incomprehensible. It should have appeared like this:

Notch filter derivation: Top circuit:

$$\frac{V_{in} + V_2}{2} = \frac{sCR}{1 + sCR} V_{in}$$

$$\text{so } V_2 = -V_{in} \left(\frac{1 - sCR}{1 + sCR} \right)$$

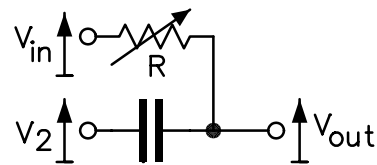


Bottom circuit:

$$V_{out} = \frac{V_{in}/R + V_2 sC}{1/R + sC}$$

$$\text{gives } V_{out} = V_{in} \left(\frac{1 - \omega^2 C^2 R^2}{1 - \omega^2 C^2 R^2 + 2j\omega CR} \right)$$

$$\text{so null when } f = \frac{1}{2\pi RC}$$



Expanded derivation.

Since the output impedance of the operational amplifier circuit will be low (less than an ohm), and we assume an ideal, zero-impedance signal source, the circuit can be analysed in two sections. The first section, shown at top in figure 1 produces a voltage V_2 which is then combined with the input voltage, V_{in} by the RC network shown at the bottom. Thus V_{in} feeds *both* sections.

To analyse the top section, we apply the standard rules obeyed in any configuration where an ideal operational amplifier is operating in linear mode. The output voltage always varies to keep both input terminals at the same potential, and no current flows into either input. Since both resistors labelled R_1 have the same value, the voltage at their midpoint, V_- , will be the mean of the voltages to which the other ends of the resistors are connected, which are V_{in} and V_{out} .

$$V_- = \frac{V_{in} + V_2}{2} \quad (1)$$

By the voltage division theorem, the voltage at the non-inverting input will be

$$V_+ = V_{in} \left(\frac{R}{R + \frac{1}{sC}} \right) = V_{in} \left(\frac{sCR}{sCR + 1} \right) \quad (2)$$

$$\text{where } s = \text{the standard abbreviation for } j\omega, \quad (3)$$

$$j = \text{the complex "quadrature" operator,} \quad (4)$$

$$\omega = \text{the angular frequency} = 2\pi f \quad (5)$$

$$\text{equating equations 1 and 2, } \frac{V_{in} + V_2}{2} = V_{in} \left(\frac{sCR}{sCR + 1} \right) \quad (6)$$

$$\text{rearranging and simplifying, } V_2 = -V_{in} \left(\frac{1 - sCR}{1 + sCR} \right) \quad (7)$$

V_{in} and V_2 are combined to give the output voltage, V_{out} , with the RC network. This is most readily analysed using Millman's theorem, which is not widely known. Figure 2 shows a network of any number of ideal voltage sources, each in series with an impedance, connected between two nodes, A and B . Any number of ideal current sources and single impedances (linear sources) can also be connected between the nodes. For convenience, the impedances are labelled by their *admittance* values (reciprocal of the impedances.)

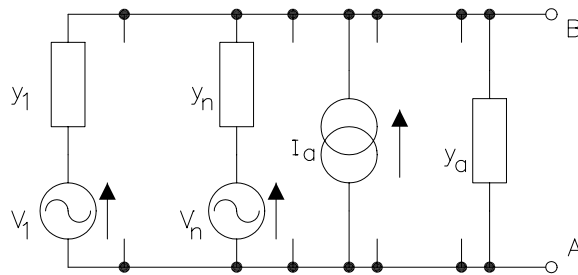


Figure 2: General circuit to illustrate Millman's Theorem.

Millman's theorem allows one to write down the resulting voltage between the two nodes by inspection!

$$V_{BA} = \frac{V_1 y_1 + V_2 y_2 + \dots + V_n y_n + I_a + \dots + I_k}{y_1 + y_2 + \dots + y_a + \dots + y_m} \quad (8)$$

This is readily proved by transforming the linear voltage sources into current sources, adding the currents, and adding the admittances to obtain a single current source. The result is then obvious. ¹

¹proved in *Linear Steady-state Network Theory*, Bold and Earnshaw, page 45. Obtainable for \$15 from the author, or the Physics Department, University of Auckland.

Applying this theorem to the combining network, where $V_{BA} = V_{out}$,

$$V_{out} = \frac{V_{in}/R + V_2 sC}{1/R + sC} \quad (9)$$

Since we require a *frequency dependent* condition, we set $s = j\omega$ and $s^2 = -\omega^2$ (since $j^2 = -1$).

$$\text{substituting, } V_{out} = V_{in} \left(\frac{1 - \omega^2 C^2 R^2}{1 - \omega^2 C^2 R^2 + 2j\omega CR} \right) \quad (10)$$

V_{out} will be zero when the numerator is zero:

$$1 - \omega^2 C^2 R^2 = 0 \quad (11)$$

$$\text{or } \omega = \frac{1}{RC} \quad (12)$$

$$\text{or when } f = \frac{1}{2\pi RC} \quad (13)$$

This looks fine, but there are two drawbacks.

- We have assumed that *both* resistors R , and both capacitors C have exactly the same values. This requires careful capacitor matching, and a very accurate dual-ganged potentiometer. More detailed analysis shows that as these R and C values depart from equality, the null depth decreases.
- The analysis assumes that the output is unloaded. If a finite impedance is connected across the output, the null broadens. Thus a voltage follower circuit is ideally required to isolate the output from any load.

A Better Circuit

Figure 3 shows a circuit which does not suffer from *either* of these drawbacks. I first encountered it in the schematic of the Atlas 350 XL transceiver.

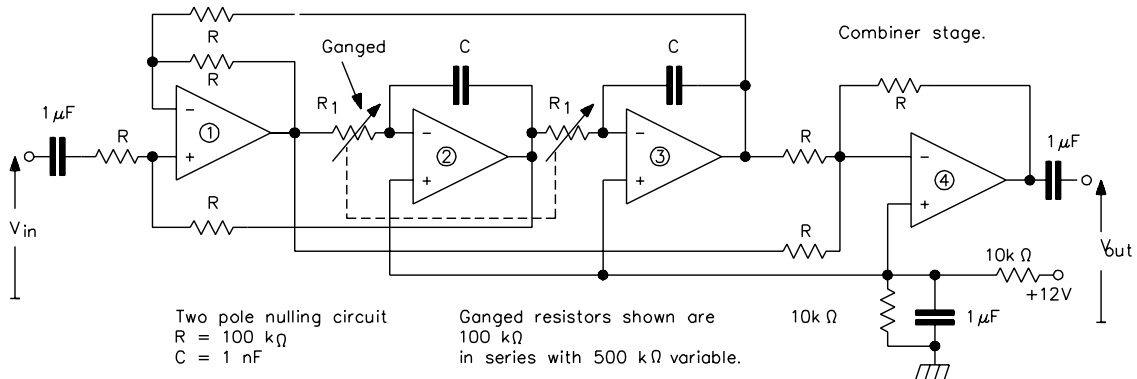


Figure 3: A better null circuit.

The null frequency is $f = 1/2\pi R_1 C$. But now variations in equality of the capacitors C and the ganged resistors R_1 only affect the *frequency* of the null, and not its depth. Since the output is taken directly from a (low impedance) operational amplifier's output, there is no loading effect.