

PI-Network Measurement of the Load Resonance Frequency without Load Capacitor

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Introduction

The measurement of quartz crystal units with load capacitance is probably the subjects that still causes most of the troubles in measurement correlation between manufacturers and users. While the PI-Network Method according to IEC 444 is generally accepted, the measurement of load resonance parameters remains to be unsatisfactory, and therefore the relevant IEC-Report 444-4 did not find general acknowledgement. Previously published methods which calculate f_L from the measured motional capacitance, show high errors due to their high sensitivity to the error of the measured C_1 and C_0 values.

The new procedure described below is based on the PI-network method of IEC 444. Instead of using physical load capacitors, this software-assisted approach determines the load resonance frequency f_L and load resonance resistance R_L from a measurement at two frequencies - one at the resonance frequency f_r and one off resonance. Once per measurement batch a calibration with open and short circuit has to be performed. (11)

The applicable frequency range is up to 125 MHz, i.e. the same as that of IEC 444. Basically this method can also be adopted by other measurement techniques such as the s-parameter method of EIA-512.

1. Principle of measurement

1.1 Fundamentals

If a crystal unit with a series resonance frequency f_s and the parameters R_1 , C_1 resp. L_1 and C_0 is connected with a load capacitor C_L in series to it, the resulting two-pole network behaves in the same way as a crystal unit with a higher frequency, the so-called load resonance frequency f_L

$$f_L = f_s \sqrt{1 + \frac{C_1}{C_0 + C_L}} = f_s \left(1 + \frac{C_1}{2(C_0 + C_L)}\right) \quad [1]$$

a higher resonance resistance R_L

$$R_L = R_1 \left(1 + \frac{C_0}{C_L}\right) \quad [2]$$

as well as modified motional and static parameters L_1' , C_1' , C_0'

$$L_1' = L_1 \left(1 + \frac{C_0}{C_L}\right) \quad [3], [4], [5]$$

$$C_1' = \frac{C_1}{\left(1 + \frac{C_0}{C_L}\right) \cdot \left(1 + \frac{C_1}{C_0 + C_L}\right)} = \frac{C_1}{\left(1 + \frac{C_0}{C_L}\right)}$$

$$C_0' = \frac{C_0 \cdot C_L}{C_0 + C_L}$$

Therefore¹, at the load resonance frequency f_L the crystal unit shows an impedance

$$Z_L = R_L + j \cdot X_L \quad [6a]$$

$$X_L = 1/(w_L \cdot C_L) \quad [6b]$$

whose magnitude and phase are:

$$|Z_L| = \sqrt{(R_L^2 + X_L^2)} \quad [7]$$

$$\arccos(Z_L) = \arccos(X_L/R_L) \quad [8]$$

If the crystal is inserted in a PI-network according to IEC 444, the effective impedance Z_{eff} consists of Z_L plus the PI-network source and load impedance "seen" by the crystal, which is nominally $R_T = 25 \Omega$:

$$Z_{eff} = Z_L + R_T = R_L + R_T + j \cdot X_L \quad [9]$$

Magnitude and phase of Z_{eff} are

$$|Z_{eff}| = \sqrt{(R_L + R_T)^2 + X_L^2} \quad [10]$$

$$\arccos(Z_L) = \arccos(X_L/(R_L + R_T)) \quad [11]$$

The output voltage U_b of the PI-network and the impedance Z_X of the inserted crystal are generally related by the equation

$$Z_X = R_T \cdot (U_{bk}/U_b - 1) \quad [12]$$

where U_{bk} is the PI-network output voltage with a short circuit instead of the crystal. The complex voltage ratio U_{bk}/U_b can be broken down into magnitude and phase ϕ_π as measured with the network analyzer or the vector voltmeter:

$$|U_{bk}/U_b| = \sqrt{((R_L + R_T)^2 + X_L^2)} / R_T \quad [13]$$

$$\phi_\pi = -\arccos(X_L/(R_L + R_T)) \quad [14]$$

1.2 Evaluation of the new method

In the following text we have considered typical quartz crystal units in the fundamental mode at 1 MHz, 3 MHz, 5 MHz, 10 MHz, 15 MHz, 20 MHz, 30 MHz, and 40 MHz as well as third overtone crystals at 30 MHz, 50 MHz, 70 MHz, and 90 MHz. The assumed motional and static parameters are given in Table 1. They are typical values for AT-cut crystals in the standard enclosure families HC-51/U, HC-49/U, and HC-52/U.

f_L [MHz]	overt	R_1 [Ohm]	C_1 [pF]	C_0 [pF]
1	1	100	10	3.5
3	1	50	8	4
5	1	30	12	4.5
10	1	15	25	5.5
15	1	10	25	6
20	1	8	25	6
25	1	5	25	6
30	1	5	25	6
40	1	5	20	5.5
30	3	30	1.8	4
50	3	25	2.2	5
70	3	20	2.2	5
90	3	20	2	5

Table 1: Parameters of evaluated quartz crystal units

1.2.1 PI-network output amplitude and phase

Figure 1a shows the relative output amplitude $|U_b/U_{bk}|$ of the PI-network as a function of the load capacitance C_L for typical fundamental mode quartz crystal units from 1 MHz to 40 MHz. Because of the high crystal impedance at f_s - especially for small load capacitances and low frequencies - the amplitude drops to a rather low value for frequencies below 5 MHz. The amplitude drops to a rather low value for frequencies below 5 MHz. The output amplitude for $C_L = 10$ pF...100 pF is 36 dB...56 dB below the short-circuit voltage U_{bk} . This critical range is depicted in magnified form in Fig. 1b. Above 10 MHz the attenuation of U_b with respect to U_{bk} is always less than 20 dB.

Fig. 1c displays the amplitude ratio for the 3rd overtone crystals 30 MHz ... 90 MHz. In this range, the amplitude is always sufficiently high.

Figure 2 displays the PI-network phase shift (referred to short-circuit) at f_L for fundamental mode and 3rd overtone crystals respectively. For the crystals below 20 MHz the phase is mainly in the range between -80° ... -90° for $C_L = 5$ pF...50 pF. For the overtone crystals the phase varies within a range of -40° ... -80° .

1.2.2. Crystal drive level (power and current)

At the load resonance frequency the crystal possesses an impedance, which is essentially a reactance with a small resistive part. Therefore the voltage drop across the crystal is mainly reactive, and consists only of a small dissipative portion. Thus a higher input level at the PI-network is necessary than for the measurement of f_L to achieve the same crystal drive level (i.e. dissipated power).

From the short-circuit output voltage U_{bk} of the PI-network the crystal drive level P_q and the crystal current I_q can be computed as follows:

$$P_q = 4 \cdot U_{bk}^2 \cdot R_L / (k_\pi^2 \cdot ((R_L + R_T)^2 + X_L^2)) \quad [15]$$

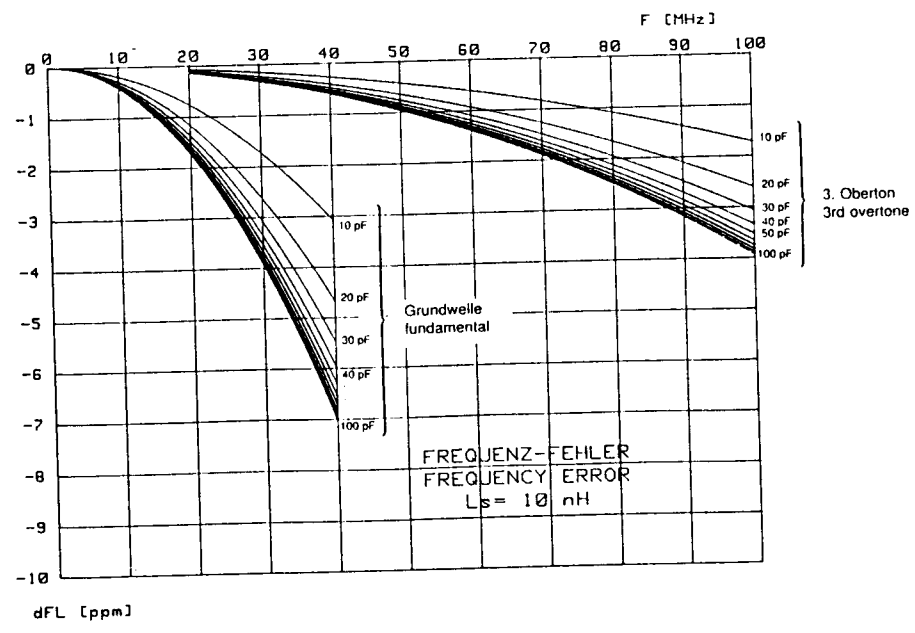
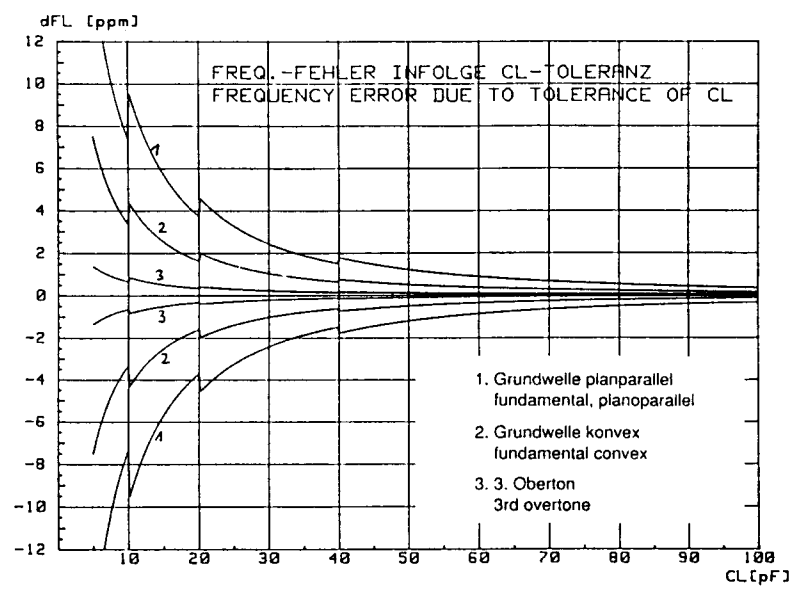
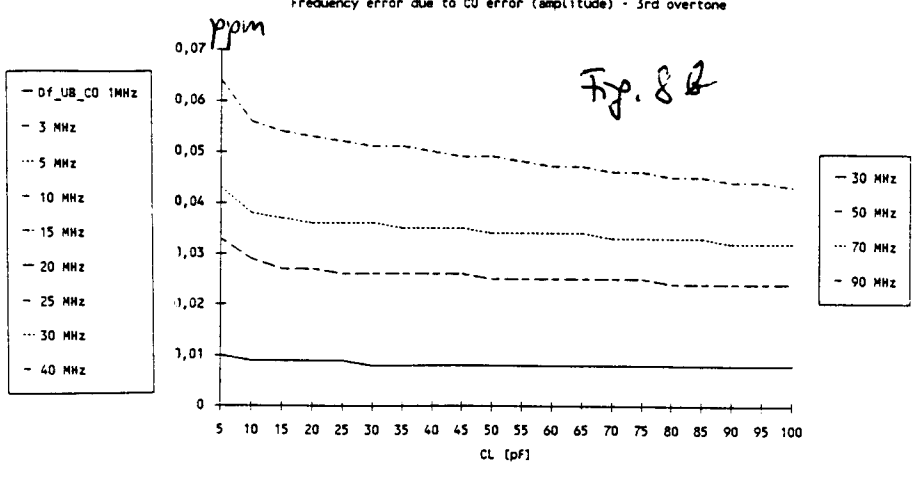
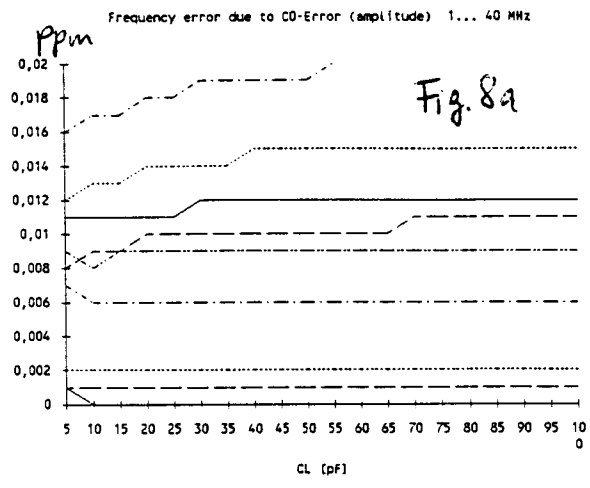
$$I_q = 2 \cdot U_{bk} / (k_\pi \cdot \sqrt{((R_L + R_T)^2 + X_L^2)}) \quad [16]$$

where

$$k_\pi = 0.3649166$$

$$X_L = 1/(w_L \cdot C_L)$$

$$R_T = 25 \Omega$$



For a specified drive level P_q resp. I_q the voltage U_{bk} has to be adjusted to

$$U_{bk} = k_{\pi}/2 * \sqrt{(P_q * ((R_L + R_T)^2 + X_L^2)/R_L)} \quad [17]$$

$$U_{bk} = k_{\pi}/2 * I_q * \sqrt{((R_L + R_T)^2 + X_L^2)} \quad [18]$$

Figs. 3a and 3b show the crystal drive level for a PI-network input power level of +30 dBm computed from equation [15]. A crystal drive level of 100 μ W can be achieved for crystals above 10 MHz with a load capacitance of 25 pF. At lower frequencies and smaller C_L -values the maximum drive level drops down to some 10 μ W. As can be seen in Fig. 3b, the 3rd overtone crystals can be easily driven to 100 μ W and above. Applying equation [16], Fig. 4 depicts the crystal current for +30 dBm PI-network input power level. A crystal current of 1 mA can be realized for crystals above 5 MHz with a load capacitance of 25 pF and more. Again, no problem exists for overtone crystals to be operated above 1 mA.

1.2.3 Measurement sequence

If the resonance resistance R_r and the static capacitance C_0 are known, the crystal impedance Z_L at the load resonance frequency f_L can be calculated from equation [6]. With the equations [13] and [14] the appropriate voltage amplitude and phase readings on the network analyzer can be computed. Thus the load resonance can be determined by tuning the generator to that frequency above the resonance frequency f_r , at which the PI-network output voltage has the required values of amplitude and phase. As will be shown in paragraph 2, the best accuracy is achieved, if the search is performed for the voltage condition only and the phase condition is neglected (or used only for a plausibility check).

For the sequence of measurements two different generator levels are used:

- For the measurement of f_r and R_r : generator output level adjusted such, that the specified crystal drive level at phase zero is realized. Subsequently this generator level will be called "LO"-level. For a crystal drive level of P_q (mW) resp. a crystal current I_q (mA), the PI-network output voltage U_{bkLO} with short circuit inserted is (for a 25 Ω crystal):

$$U_{bkLO} = 57.8 \text{ mV} * \sqrt{(P_q/1\text{mW})} \quad [19]$$

$$U_{bkLO} = 9.138 \text{ mV} * I_q/1\text{mA} \quad [20]$$

With the conventional IEC 444 set-up the generator level is about 35.6 dB higher than U_{bk} .

- For all other measurement: the generator level is set to the "HI"-level, at which the nominal drive level is reached at the load resonance frequency. The applicable formulas [17] and [18] are given in paragraph 1.2.2. The PI-network output voltage with short circuit is denoted as U_{bkHI} .

Based on the measurement set-up according to IEC 444-1, the complete measurement sequence is as follows:

Step 1: calibration

a) PI-network without crystal ("open circuit"): The generator output is set to the "HI"-level and is tuned to f_{CO} , the frequency at which C_0 will be measured (see step 2). The cross-talk capacitance C_c is computed from the PI-network output voltage U_{open} :

$$C_c = 1/(w_{CO} * R_T * \sqrt{(U_{bkHI}/U_{open}-1)}) \quad [21]$$

b) PI-network with short circuit: The generator is tuned to the nominal frequency f_{nom} of the load resonance. The phase reading is set to zero (reference value). To compensate possible amplitude errors of the vector voltmeter, this calibration is performed at the "HI"-level (PI-network output voltage U_{bkHI}) and at the "LO"-level (voltage U_{bkLO}). While U_{bkHI} is used as reference for the f_L measurement, U_{bkLO} is the reference for the zero phase measurement of f_r and R_r .

Step 2: Measurement of C_0

PI-network with crystal: The generator is adjusted to the "HI"-level and tuned to a frequency f_{CO} approximately 10% below f_{nom} . For nominal frequencies below 5 MHz, f_{CO} should be selected between 5 and 15 MHz to avoid noise problems at too low output voltages. Alternatively the recent suggestion of IEC TC49/WG 6 may be followed.

The static capacitance C_0 is determined from the output voltage reading U_{CO} . From the result the cross-talk capacitance has to be subtracted

$$C_0 = 1/(w_{CO} * R_T * \sqrt{(U_{bkHI}/U_{CO}-1)}) - C_c \quad [22]$$

Step 3: Measurement of f_r and R_r

PI-network with crystal: The generator is set to the "LO"-level and tuned to that frequency, where zero phase (relative to the short circuit calibration with U_{bkLO}) occurs. To speed up the search for f_r , an estimate of f_r can be computed with the help of equation [1] using an estimated value for C_1 and the measured value of C_0 .

The resonance resistance R_r can be calculated (as in IEC 444-1) from U_{bkLO} and the voltage U_b at zero phase according to equation [12].

Step 4: Measurement of f_L , determination of R_L
PI-network with crystal: From the measured R_r the load resonance resistance R_L is determined by equation [2] using the measured C_0 value.

The voltage amplitude at f_L has to be computed from equation [13] using U_{bkHI} . The load resonance frequency f_L is then found by setting the generator to the "HI"-level and tuning to that frequency above f_r , at which the amplitude condition is fulfilled. A first guess f_{L1} is the nominal value f_{nom} . With the amplitude reading at a second frequency a few ppms from f_{nom} , a good second guess f_{L2} can be computed by interpolation. Because in the vicinity of f_L the amplitude vs. frequency response is rather linear, the final value of f_L can be reached by very fast iteration. As this tuning is off resonance, the frequency stepping can be realized very fast without degradation by ringing effects of the crystal.

2. Error considerations

A detailed error analysis has been performed to consider the influence of different measurement errors. A comparison with the conventional method using a physical load capacitor is made in paragraph 2.5.

2.1 Error of the amplitude measurement

Assuming an error of 0.5% for the amplitude measurement of the network analyzer or the vector voltmeter respectively leads to an erroneous location of f_r . The hereby caused frequency error is depicted in Figs. 5a and 5b. For fundamental mode crystals the error is worst at small load capacitances for the crystals at and above 10 MHz. The maximum error is less than 3 ppm, the average is at about 1.5 ppm. Overtone crystals show a much smaller error sensitivity of less than 0.3 ppm.

2.2 Error of the phase measurement

Because of the fact that in the vicinity of f_r the phase angle is close to constantly 90°, the frequency sensitivity to phase errors is very high as shown in Fig. 6a and 6b. Therefore it is strongly recommended to use only the amplitude criterion of equation [13] and not the phase criterion of [14] for the search of f_r . The phase information should only be used as a rough indicator to verify that the measured data are regular. The following charts are all under the assumption, that the amplitude-search method is used.

2.3 Error of R_r and DLD

If the determination of R_r at zero phase is erroneous, this leads to a frequency error for f_L . For an assumed R_r -error of 1% the results are shown in Fig. 7a and 7b for the fundamental and overtone mode crystals resp. In both cases the error increases with frequency and with C_L . It is in the order of a tenth of a ppm and can therefore be neglected compared with the other influences. The same graphs can also be used for a judgement about the sensitivity of this method to drive level dependencies (DLD) inherent in real crystal resonators. A 10 MHz/ 30 pF crystal unit with a drive level dependent resonance resistance varying 10% has an additional frequency error due to this DLD effect of 0.1 ppm, which certainly can be neglected.

2.4 Error of C_0

Because C_0 is needed to compute R_L , which itself is a term in the amplitude equation [13], the measurement error of C_0 influences the frequency accuracy of f_L . Fig. 8 shows the computed frequency error for a C_0 error of 2%. For fundamental as well as 3rd overtone crystals this error is only in the order of 10^{-5} and is therefore negligible.

2.5 Comparison with the classical method

The classical method of measuring f_L uses a physical, i.e. real, load capacitor as an adaptor switched in between the measuring jig and the crystal unit under test. This is the method described in IEC report 444-4, which did not find general acknowledgement due to its insufficiencies. The achievable tolerances claimed in this document are by far not reached in practice.

More realistic figures about the possible accuracy in realizing load capacitance adaptors at different places are as follows:

C_L up to	9.9 pF	tolerance	± 0.15 pF
10 ...	19.9 pF		± 0.20 pF
20 ...	39.9 pF		± 0.25 pF
40 ...	99.9 pF		± 0.30 pF
above	100 pF		± 0.40 pF

Dividing the sample crystal types of table 1 into only three groups

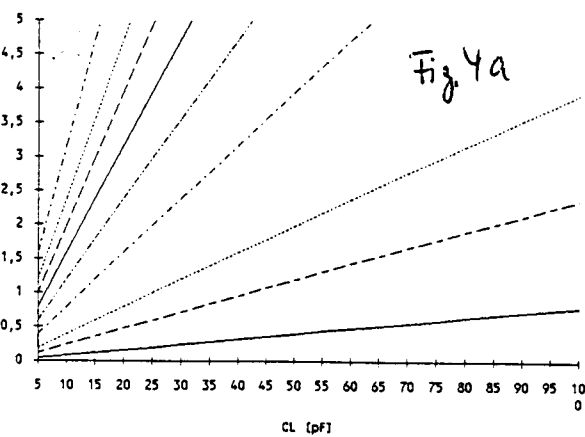
group 1	(planoparallel)	: $C_1 = 25$ fF; $C_0 = 6$ pF
group 2	(convex, contoured)	: $C_1 = 10$ fF; $C_0 = 5$ pF
group 3	(third overtone)	: $C_1 = 2.2$ fF; $C_0 = 6$ pF

the computed measurement error induced by the above mentioned tolerances of the load capacitors is depicted in Fig. 9.

Furthermore an important frequency dependent error is inherent in every load capacitor adaptor, which is caused by the inevitable inductance of the connecting leads of the adaptor. Assuming this inductance to be totally 10 nH this frequency dependent error is in the order of several ppm as depicted in Fig. 10, which has to be added to the error numbers of Fig. 9.

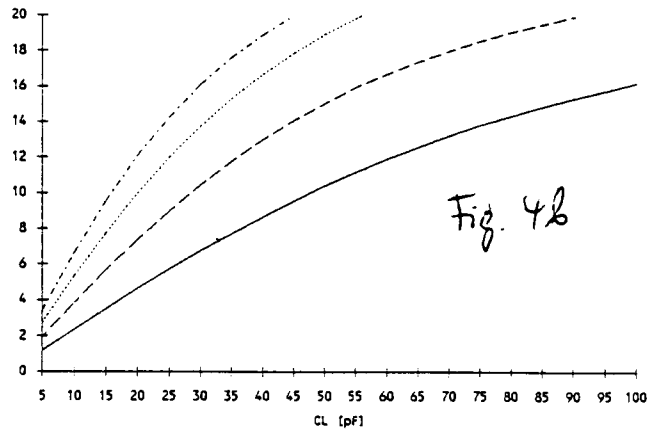
It can be seen that the frequency error of the classical method is

Crystal current for 30dBm input power level



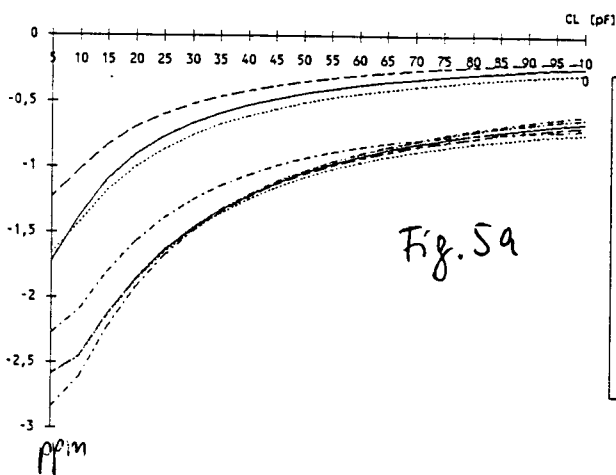
- $I_{a30} / 1\text{MHz}$
- 3 MHz
- ... 5 MHz
- 10 MHz
- 15 MHz
- 20 MHz
- 25 MHz
- ... 30 MHz
- 40 MHz

Crystal current for 30dBm input power level - 3rd overtone



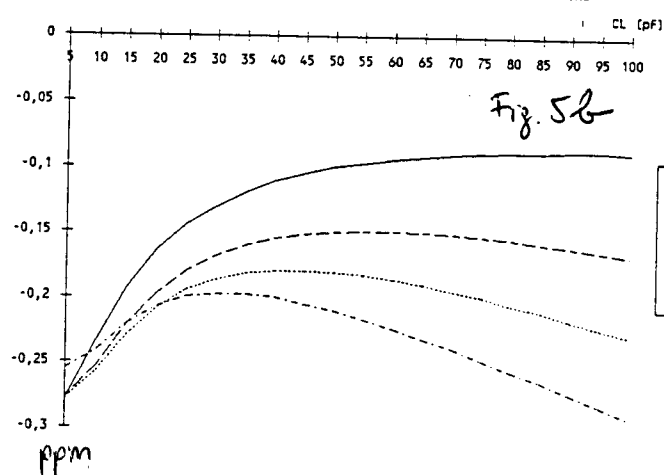
- 30 MHz
- 50 MHz
- ... 70 MHz
- 90 MHz

Frequency error due to amplitude error of 0.5% / 1 MHz ... 40 MHz



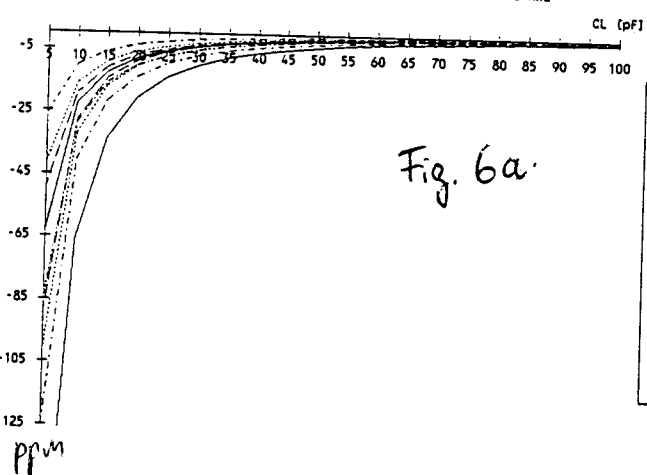
- $\Delta f_{UB} 1\text{MHz}$
- 3 MHz
- ... 5 MHz
- 10 MHz
- 15 MHz
- 20 MHz
- 25 MHz
- ... 30 MHz
- 40 MHz

Frequency error due to amplitude error of 0.5% / 3rd overtone



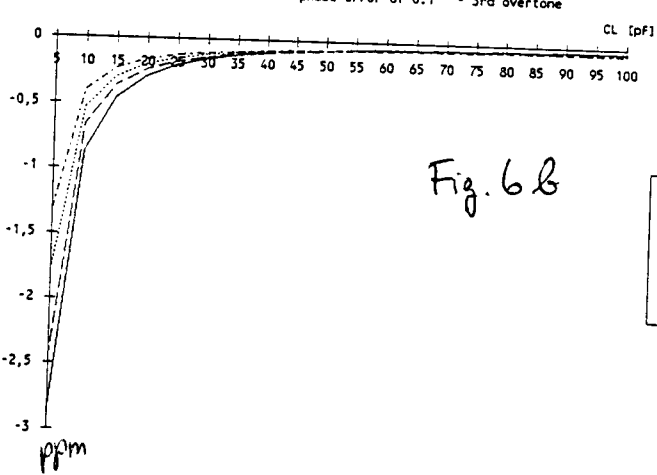
- 30 MHz
- 50 MHz
- ... 70 MHz
- 90 MHz

Frequency error due to phase error of 0.1° - 1 ... 40 MHz



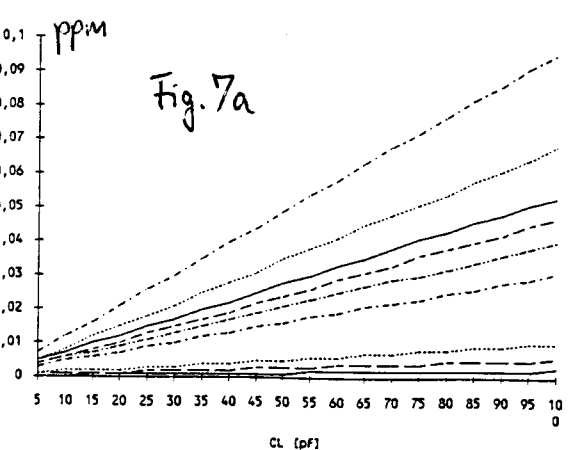
- 1 MHz
- 3 MHz
- ... 5 MHz
- 10 MHz
- 15 MHz
- 20 MHz
- 25 MHz
- ... 30 MHz
- 40 MHz

Frequency error due to phase error of 0.1° - 3rd overtone



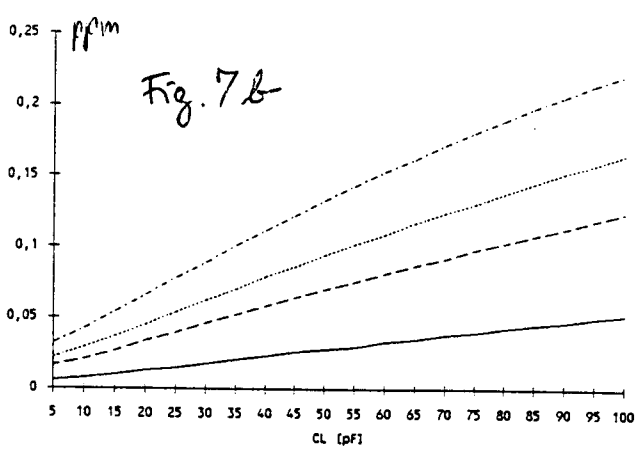
- 30 MHz
- 50 MHz
- ... 70 MHz
- 90 MHz

Frequency error due to R_r error (amplitude) 1 ... 40 MHz



- $\Delta f_{UB} R_r 1\text{MHz}$
- 3 MHz
- ... 5 MHz
- 10 MHz
- 15 MHz
- 20 MHz
- 25 MHz
- ... 30 MHz
- 40 MHz

Frequency error due to R_r error (amplitude) - 3rd overtone



- 30 MHz
- 50 MHz
- ... 70 MHz
- 90 MHz

at least three times higher than the most dominant error of the new method (Fig.5a).
An important advantage of the new method is, that it is completely software-based, and therefore independent of the mechanical construction and calibration accuracy of any C_0 -adaptor. So basically the same measurement results can be achieved at different places with different equipments, which are only limited by the accuracy of the instrumentation.

2.6 Additional aspects

2.6.1 Measurement of load resonance frequency vs. temperature

Besides the measurement at room temperature, an important factor in practice is the measurement of the load resonance frequency in the temperature range. The classical approach with load capacitor adaptors yields several problems in the practical application, as it is necessary to place the capacitors outside the temperature chamber, which causes considerable problems with lead length etc. Measuring crystals with different load capacitance at the same temperature run is also difficult to handle etc. The new method allows to measure the f_L vs. temperature characteristics easily without hardware problems. Because the crystal is measured directly at the same operating point, where it works with the load capacitance in the final circuit, all changes of this point in the temperature range can be tracked, the regular $f_L(\theta)$ response as well as irregularities due to activity dips, spurious responses etc.

Existing software methods⁴ which compute the load resonance frequency from the measurement of f_r , C_1 and C_0 do not yield the exact $f_L(\theta)$ response (which is different to the $f_r(\theta)$ response), and cannot detect irregularities exactly, because they occur differently at f_r and f_L .

2.6.2 Application to error-corrected measurement techniques

Further improvements can be achieved, if the PI-network is used together with an error correction technique as described in several publications^{5,6,7,8}. The algorithm described in this paper can be implemented easily into that method. As the error corrected PI-network technique is basically an impedance measurement method, the load resonance frequency can be found at that frequency, where the crystal impedance magnitude has the value given in equation [10]. With error correction the measurement results will be nearly completely independent of the used measurement jig (PI-network or others). This would e.g. allow to replace the IEC PI-network by a modified construction with lower attenuation to overcome limitations regarding the maximum applicable drive level at lower frequencies. In the same way an implementation into other impedance measurement methods such as the s-parameter technique^{9,10} can be performed, which actually do not offer any load resonance measurement feature.

3. Summary

A new software-assisted measurement method for the load resonance of crystal units based on the PI-network method of IEC 444 could be presented. The article describes in detail the fundamentals as well as all equations necessary for the implementation of the method. It includes the results of a detailed error analysis, which shows the superiority of the new method over the existing classical method of IEC 444-4 and over previously published methods, which calculate the load resonance from normal resonance frequency measurements.

The method will be included in a forthcoming proposal for IEC.

¹ In the following formulas, the series resonance frequency f_s is replaced by the resonance frequency f_r (at phase zero). This approximation is sufficiently accurate in the considered frequency range.

² An optimum choice for f_{r0} is given in a proposal agreed at the IEC TC49 / WG 6 meeting (Brighton, July 1989) as follows:

For crystals up to 30 MHz, the measurement is made at 5 frequencies slightly above 30 MHz; i.e. 30.1, 30.2, 30.3, 30.4, and 30.5 MHz; and the average of the three values nearest the mean is used as the best estimate.

For crystals above 30 MHz, measurements are recommended at three paired frequencies, each equidistant from f_s , i.e. $f_s \pm (1 \pm 0.05)$, $f_s \pm (1 \pm 0.06)$, and $f_s \pm (1 \pm 0.07)$. For each pair, the value of C_0 is calculated of the lower and higher frequency, and a value of $C_0(i)$ is obtained as the mean of the two C_0 values. This gives three estimates, $C_0(i)$, $i=1,2,3$. The best value to be used for C_0 is the average of the two values which are closest to each other.

In either case, care should be given that the measurement frequencies are not coincidental with an unwanted mode of vibration

³ Neubig, B.W.: Measurement of Quartz Crystal Units with Load Capacitance; TELE QUARZ GmbH, D-6924 Neckarbischofsheim 2 (W.-Germany), Extra print 10/89

⁴ Sauerland, F.+ Saunders, J.: General Xtal Test Methods Related To Oscillators and Transmission Instruments; Proc. 3rd Quartz Crystal Conference, Kansas City (Aug. 1981), p.210-228

⁵ Neubig, B.W.: Measurement of Quartz Crystal Units up to 500 MHz and Above by the Use of a PI-Network with Error Correction; Proc. 11th Quartz Devices Conference, Kansas City (Aug.1989), Vol.1, p.60 ff.

⁶ Neubig, B.W.: Fortschritte auf dem Gebiet der Schwingquarz-Meßtechnik bis 500 MHz; Tagungsband ZVEI-Symposium Schwingquarze Frankfurt/M. (14.6.1989), pp.106-126

⁷ Neubig, B.W.+ Zimmermann, R.: Recent Advances on the Measurement of Quartz Crystal Units up to 500 MHz and Above by the Use of a PI-network with Error Correction; Proc. 2nd International Conf. on Frequency control and Synthesis; Leicester/UK (April 1989), p. 11 ff; IEE Conference Publ.No.303

⁹ Peach, R.C. + Morris, S.E.: A System for Precision Measurement on Quartz Crystal Resonators And Bipoles; Proc. 39th Annual Symposium on Frequency Control Philadelphia/USA (1985), pp. 527-534

¹⁰ EIA-512: Standard Methods for Measurement of the Equivalent Electrical Parameters of Quartz Crystal Units, 1kHz to 1 GHz; Electronic Industries Association, Washington/USA (1985)

¹¹ At the presentation of the paper during the 4th EFTF we had to learn from R.C.Smythe that the idea was already suggested earlier in:

Horton,W.H., Payne,T.S., Smythe,R.C. + Symonds,D.A.: Comparison of Methods for Measurement of Quartz Crystal Resonators with Load Capacitance; Proc. 35th Annual Symposium on Frequency Control Philadelphia/USA (1981), pp.271-279. Nevertheless, the detailed description suitable for direct application to the PI-Network method as well as the extensive error analysis in our actual paper are unique.

