These expressions are identical to those for the lossy equal-element band-pass filter, as given in (B-9) of reference [4], except that $1/X$ replaces the X used in the band-pass case.

Expressions for any value of n can be determined in a manner similar to that given in (B-11) of reference [4]. The result for the band-stop case is

$$|M_n|^2 = \left| \sum \left[\binom{n-s}{s} \frac{1}{X^{n-2s}} + \binom{n-2-s}{s} \frac{1}{X^{n-2-2s}} + 2\binom{n-1-s}{s} \frac{1}{X^{n-1-2s}} \right] \right|^2$$

where the summation of s includes only terms where $n-2s$, $n-2-2s$, $n-1-2s$ are all equal to, or greater than, zero. The terms within the parentheses are binomial coefficients, tables of which have been compiled by the Smithsonian Institute [10].

ACKNOWLEDGMENT

The authors are grateful to V. Lee of Airborne Instruments Laboratory, Cutler-Hammer, Inc., for having performed the numerical calculations.

REFERENCES

[1] L. Young, G. L. Matthaei, and E. M. T. Jones, "Microwave band-stop filters with narrow stop bands," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-10, pp. 416–427, November 1962.

[2] B. M. Schiffman and G. L. Matthaei, "Exact design of band-stop microwave filters," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 6–15, January 1964.

[3] S. B. Cohn, "Dissipation loss in multiple-coupled resonator filters," *Proc. IRE*, vol. 47, pp. 1342–1348, August 1959.

[4] J. J. Taub, "Design of minimum loss band-pass filters," *Microwave J.*, vol. 6, pp. 67–76, November 1963.

[5] L. Young, "Group delay and dissipation loss in transmission-line filters," *IEEE Trans. on Microwave Theory and Techniques* (*Correspondence*), vol. MTT-11, pp. 215–217; May 1963.

[6] H. A. Bethe, "Lumped constants for small irises," MIT Radiation Labs., Cambridge, Mass., Rept. 43-22, March 1943.

[7] S. B. Cohn, "Determination of aperture parameters by electrolytic-tank measurements," *Proc. IRE*, vol. 39, pp. 1416–1421, November 1951.

[8] S. B. Cohn, "Microwave coupling by large apertures," *Proc. IRE*, vol. 40, pp. 696–699, June 1952.

[9] L. Storch, "The transmission matrix of N alike cascaded networks," *Trans. AIEE* (*Communications and Electronics*), vol. 73, pp. 616–618, 1954 (January 1955 Sect.)

[10] *Smithsonian Mathematical Formulae and Tables of Elliptic Functions*. Washington, D. C.: Smithsonian Institute, 1922, p. 20.

YIG Filters as Envelope Limiters

R. C. CUMMING, SENIOR MEMBER, IEEE, AND D. W. HOWELL, STUDENT MEMBER, IEEE

Abstract—Envelope limiters are used in such applications as FM demodulation and power leveling. Recently, the envelope-limiting properties of yttrium-iron-garnet (YIG) filters were reported for the special cases of unmodulated and pulsed input signals. Measured data is presented here on the response of a YIG limiter to AM carriers having modulation index of the order of 50 percent. Sinusoidal, square-wave, and low-pass noise modulating signals were used in the measurements. It was found that a YIG filter will give good envelope limiting for modulating frequencies in the submegacycle range. At these low frequencies the carrier and the side frequencies are not limited selectively. At higher modulating frequencies where the limiting is frequency selective, the YIG filter will not remove the envelope variations. In fact, in the particular filter tested, the modulation index was increased, rather than decreased, at modulating frequencies greater than about 750 kc/s. A graph is given showing the measured factor of reduction (or increase) of modulation index, as a function of modulating frequency. The response of the limiter as a function of carrier frequency, modulating frequency, and input power is shown by oscilloscope displays produced by sweeping the carrier frequency or input power. In addition, selected photographs of output envelope waveforms are given.

INTRODUCTION

THE APPLICATION of a YIG filter as a conventional bandpass limiter, i.e., the kind of limiter that removes envelope variations from a narrowband signal, is presented. This *envelope limiter* should be distinguished from the frequency-selective type where each frequency component in the signal is limited independently [1], [2]. Perhaps the most familiar application of envelope limiters is in FM demodulators, where they are used to prevent envelope variations from appearing in the output. Another application is in power leveling of signal generators.

The general conclusion of our work is that a YIG filter will function as an envelope limiter for relatively slow envelope variations, and as a frequency-selective limiter for relatively fast envelope variations. In terms of sinusoidal amplitude modulation, when the modulating frequency is low relative to the decay time constant of the spin-wave modes of the YIG, the filter acts as an envelope limiter. When the modulating frequency is high relative to that time constant, the device acts as a frequency-selective limiter.

The envelope-limiting properties of YIG filters have

Manuscript received April 1, 1965; revised June 4, 1965. This paper is based on work sponsored by the Air Force Avionics Lab., Wright-Patterson Air Force Base, Ohio, under Contract AF 33(657)-11144.

The authors are with Electronics Laboratories, Stanford University, Stanford, Calif.

recently been investigated and reported for two major classes of input signals: unmodulated signals and pulsed signals [3]–[6]. In this paper, the limiting properties of a particular YIG filter are discussed when the input signal is amplitude-modulated, with a modulation index on the order of 0.5, and a minimum envelope power that is greater than the limiting threshold power of the YIG. With this type of signal, the limiter is always operating in its saturation region, and the mode of response might be expected to be something between those of the unmodulated and pulsed signals. The particular amplitude modulations used in the response measurements were 1) 25- and 50-percent sinusoidal modulation, with various modulating frequencies, ranging from 10 kc/s to 10 Mc/s, 2) 25- and 65-percent square-wave modulation, and 3) low-pass noise modulation.

The YIG limiter used in all the measurements was a Physical Electronics Laboratories Model LS-101, which is a coincidence-type limiter having the following specifications:

Resonant frequency f_0	2.86 Gc/s
Small-signal 3-dB bandwidth	18 Mc/s
Small-signal insertion loss	1.2 dB
Limiting level at f_0	−10 dBm
Dynamic limiting range at f_0	30 dB

With pure YIG, coincidence-type limiters can be designed to operate anywhere within the approximate frequency range 1800 Mc/s to 3400 Mc/s. The definition and theory of operation of coincidence and other types of ferrimagnetic limiters are readily available in the literature, as is information on other ferrimagnetic materials and their frequency ranges [3]–[5], [7], [8].

MEASUREMENTS

A. *Unmodulated Input*

For reference, the unmodulated limiting characteristics of the YIG limiter are presented in Fig. 1(a), which shows that limiting begins at about −10 dBm and that the output increases only slightly as the input power is increased by 30 dB. An idealized three-dimensional graph of this data, showing detected output amplitude as a function of input power and frequency, is given in Fig. 1(b). Note that the bandwidth over which the filter acts as a limiter is an increasing function of input power. Or, stated conversely, the threshold power input required for limiting increases as the frequency departs from the center frequency of the filter. This behavior is characteristic of ferrimagnetic and varactor-diode parametric limiters [6].

B. *Response vs. Carrier Frequency; Sinusoidal Modulation*

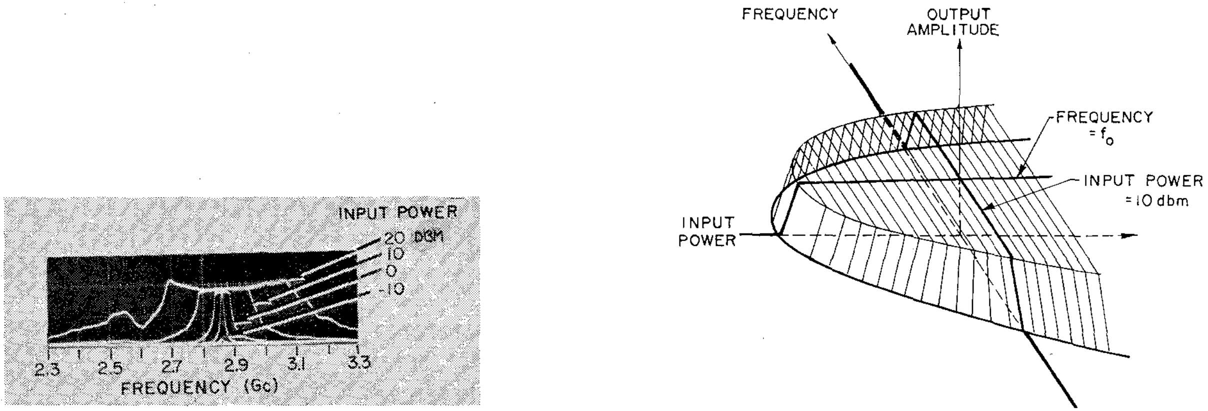
Figure 2 is a series of frequency-response curves, made in the usual way using a signal generator whose frequency is slowly swept in synchronism with the horizontal sweep of the oscilloscope. However, instead of holding the input signal level constant, the input was purposely amplitude modulated. The vertical deflection of the oscillograph spot was produced by the output of an envelope detector. A dual-trace oscilloscope was used, the upper trace showing the input signal and the lower trace showing the output signal. The vertical width of the "trace" is indicative of modulation percentage. In the upper trace, the vertical width is nearly independent of the horizontal coordinate, showing the input modulation percentage to be nearly constant, independent of carrier frequency. In each photograph in Fig. 2, the carrier power and modulation index were held nearly constant at +10 dBm, and 50 percent, respectively. Sinusoidal modulation was used, with various modulating frequencies as indicated on the photographs. For reference, a photograph taken with no modulation on the input is shown; this photograph is essentially identical with the one labeled $P_{\text{in}} = +10$ dBm in Fig. 1(a). The frequency sweep rate of the signal generator was made very slow (one second per sweep), so that any transients introduced by sweeping were completely negligible. Note that the oscilloscope sensitivity was reduced in the third row of photographs.

The lower halves of each photograph in Fig. 2 show how the modulation index (vertical width) varies with carrier frequency (horizontal coordinate). The degree of output AM depends on both the modulating frequency and the carrier frequency. At relatively low modulating frequencies, the YIG filter appears to function quite well as an envelope limiter, except near the edges of the pass band, where the carrier power is close to the limiting threshold of the device. (As mentioned before, the limiting threshold is about −10 dBm at band center, and increases as the frequency departs from band center.) As the modulating frequency is increased, the quality of limiting deteriorates across the entire pass band of the device, and a large amount of AM appears in the output.

C. *Response vs. Input Power; Sinusoidal Modulation*

The response of the limiter was also observed as a function of the input power level. This was accomplished by varying the amplitude of the carrier signal generator (fixed frequency) by applying the oscilloscope sweep voltage to the external AM input (see Fig. 3). Sinusoidal AM was again applied to the signal, using a PIN-diode modulator. The carrier power at the end of the oscilloscope sweep was measured with a hp-431A power meter.

The experimental response data are shown in Fig. 4. Again, as in Fig. 2, the vertical deflection of the oscillograph spot is produced by the detected output envelope of the YIG filter. The input signal is not shown in this set of photographs. For all measurements in Fig. 4, the carrier frequency was centered in the filter pass band, and the carrier power was varied almost linearly from zero to the value shown at the top of each column. The horizontal sweep time was five seconds.



(a) (b)

Fig. 1. Response of YIG limiter to an unmodulated signal. (a) Measured frequency response at various input power levels. (b) Idealized three-dimensional graph of output amplitude vs. input power and frequency.

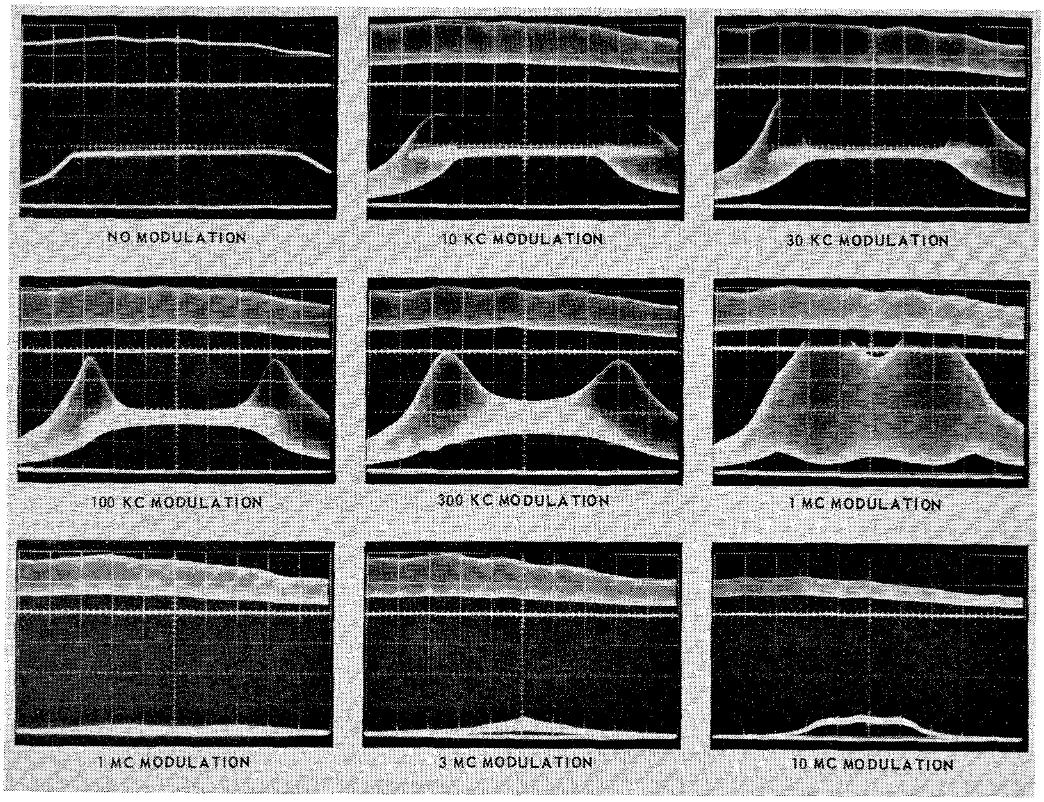


Fig. 2. Response vs. carrier frequency. Horizontal scales indicate carrier frequency, at 20 Mc/div.; $f = 2860$ Mc at center of each photograph. Vertical scales: upper trace = 50 mV/div.; lower trace = 50 mV/div. for first two rows, 5 mV/div. for third row.

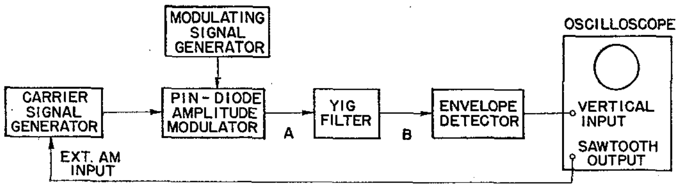


Fig. 3. Basic equipment used for measuring response vs. input power.

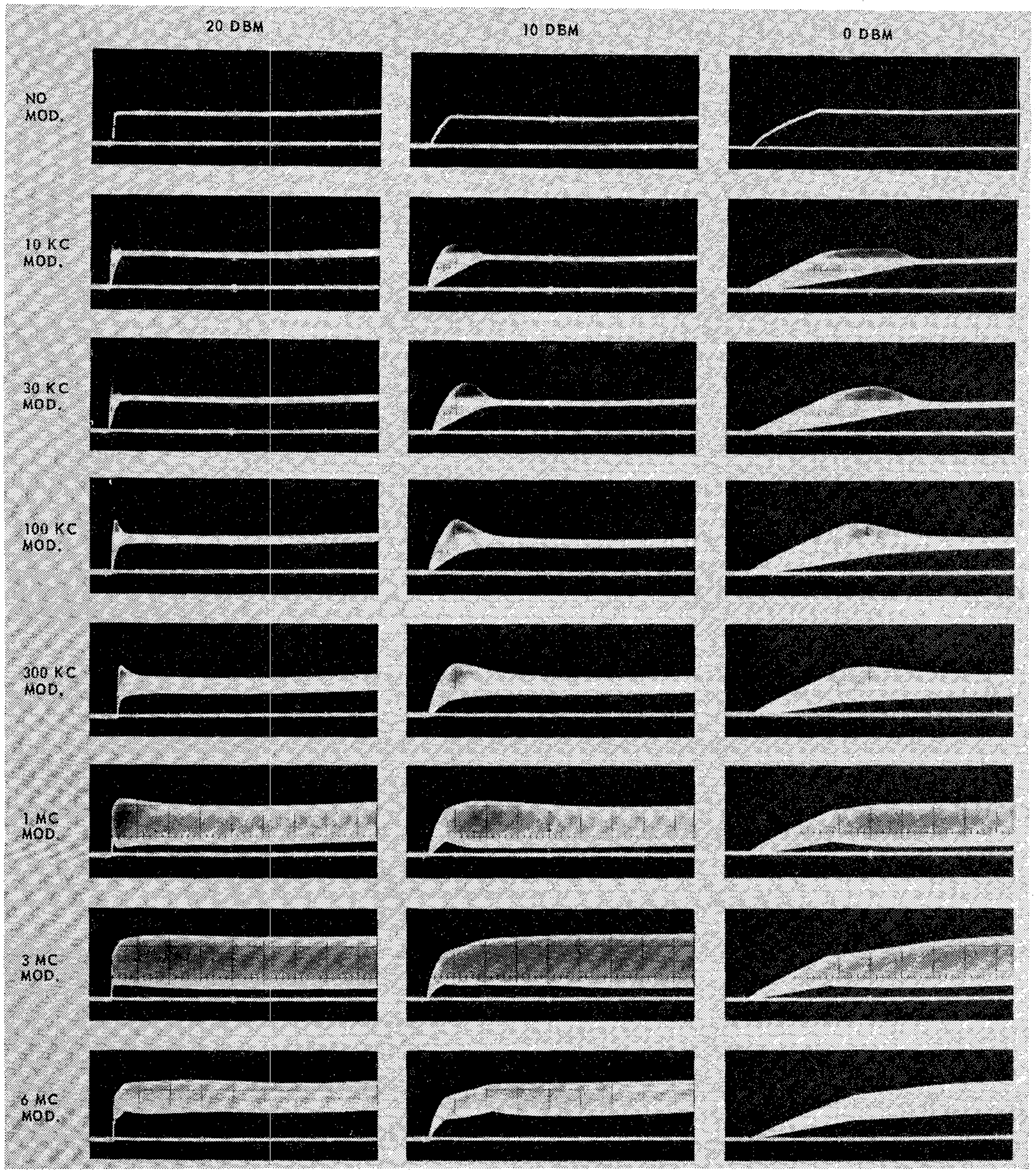


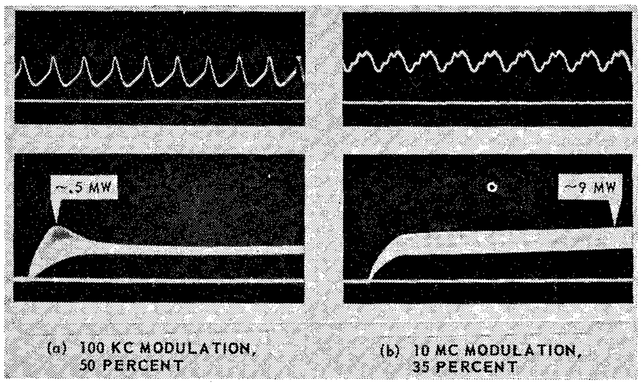
Fig. 4. Response vs. input power. Horizontal coordinate is input power which increases from zero to the value shown at the top of each column.

Various modulating frequencies were used, as indicated on the figure, and the input modulation index was adjusted to 50 percent in each case.

The main effect to be noted in the photographs is the variation of the output modulation index with modulating frequency. Below about 750 kc/s, the output modulation index is lower than that of the input; at higher modulating frequencies, the modulation index of the output actually exceeds that of the input. Note, in addition, that at any one modulating frequency the output modulation index remains very nearly constant when the input power is above the limiting threshold. Also interesting is the behavior in the vicinity of the limiting threshold. The 10-, 30-, 100-, and 300-kc/s photographs show an increase of envelope amplitude as the limiting threshold is approached. On the other hand, at modulating frequencies higher than approximately 1 Mc/s, distinct "corners" exist in the threshold region, corresponding to the power levels at which the carrier and sideband frequency components reach the limiting threshold and become limited selectively.

D. Response Waveforms; Sinusoidal Modulation

The intensity variations exhibited in Figs. 2 and 4 along vertical lines are indicative of the waveforms of the output envelope. Qualitatively, the display intensity as a function of the vertical coordinate gives the amplitude density function of the output envelope waveform. The displays of Figs. 2 and 4 indicate that, at lower modulating frequencies, the envelope waveform becomes highly distorted as the input carrier power goes through the threshold-of-limiting region. This is true, as can be seen from the 100-kc-modulation envelope waveform of Fig. 5(a). For reference, the corresponding display of response vs. input power is also shown with an arrow indicating the power level at which the waveform photo-



(a) 100 KC MODULATION, 50 PERCENT (b) 10 MC MODULATION, 35 PERCENT

Fig. 5. Output envelope waveforms, for sinusoidal input modulation.

graph was taken. At higher (and lower) power levels the envelope waveform once again becomes sinusoidal. The waveform of Fig. 5(b) is interesting because it illustrates envelope waveform distortion occurring at high modulating frequencies where the carrier and side frequencies are limited selectively. The ripples in the waveform will be seen to correspond to the density bars in the lower display.

E. *Response Waveforms; Square-Wave Modulation*

The output envelope waveform of the saturated YIG limiter in response to square-wave-modulated signals of index less than one is shown in Fig. 6. The input and output modulation envelopes are shown in the upper and lower halves, respectively, of the photographs. Minimum input levels and modulation indexes are shown on the photographs.

The response is seen to be significantly different from that obtained when the modulation index is 100 percent (ON-OFF pulsed RF). In the case of 100-percent square-wave modulation, it is well known that a "leakage spike" having a peak amplitude equal to the peak applied amplitude will be observed in the output [5], [6], [9]. With less than 100-percent modulation, however, the leakage spike in Fig. 6 is seen to have a smaller amplitude, which remains constant as the input power level is increased.

F. *Response Waveforms; Low-Pass-Noise Modulation*

The response of the limiter was also observed when the input signal was amplitude-modulated by low-pass noise. A General Radio GR-1390A noise source was used to provide 20-kc/s-, 500-kc/s-, and 5-Mc/s-bandwidth low-pass noise.

The envelope waveform of the limiter output, which was not photographed, appeared to correspond to *band-pass*, rather than low-pass noise. It was apparent, especially with 20-kc/s low-pass noise, that the low-frequency modulation components were effectively limited. It was also noticed that the amplitude of the output and its modulation index remained constant as the input power was varied.

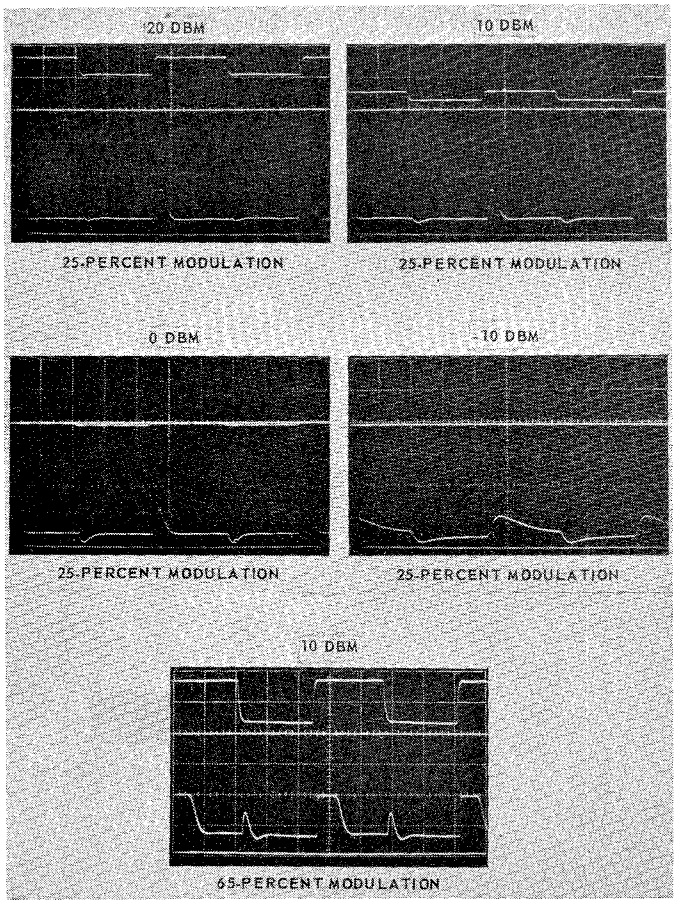


Fig. 6. Input and output envelope waveforms, for square-wave input modulation. Minimum input power and modulation indices are indicated.

G. *Modulation-Index Ratio*

One possible figure of merit for an envelope limiter is the ratio of output (amplitude) modulation index to input modulation index, with sinusoidal modulation applied to the input. In general, the output envelope will not be sinusoidal. Hence a definition of output modulation index is needed. For our purposes, we define the output (or input) modulation index as follows:

$$\text{modulation index} = \frac{\text{peak envelope voltage} - \text{minimum envelope voltage}}{\text{peak envelope voltage} + \text{minimum envelope voltage}}.$$

In YIG limiters, the modulation-index ratio varies with modulating frequency and input modulation index. However, Fig. 4 shows that the modulation-index ratio is nearly independent of input level, when the level is above the limiting threshold.

The modulation-index ratio was calculated from data measured, first using 50-percent modulation, and then using 25-percent sinusoidal modulation. For all the data, the carrier power was kept constant at 10 dBm. The input and output modulation indexes were determined using a calibrated variable attenuator at points

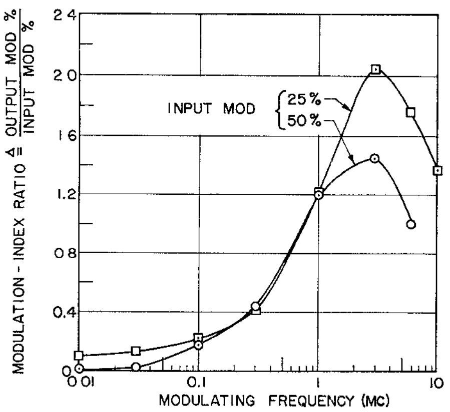


Fig. 7. Modulation-index ratio vs. modulating frequency.

A and B, respectively, in the basic system of Fig. 3. The resulting plots of modulation-index ratio vs. modulating frequency are shown in Fig. 7. The curves show that the limiter gives some reduction in modulation index at all frequencies below approximately 750 kc/s.

If the YIG filter were a perfect frequency-selective limiter at high modulating frequencies and the measuring equipment were perfect, the graph of modulation-index ratio in Fig. 7 would level off at a value of 2.0 for 50-percent modulation and 4.0 for 25-percent modulation. This leveling-off would occur because the output spectrum would consist of three equal-amplitude components whose phase relationships would be the same as those of the carrier and two side frequencies of the AM input signal. It follows that the output envelope waveform would be $|1+2\cos\omega_m t|$, which is nonsinusoidal and represents 100-percent amplitude modulation. However, the actual data plotted in Fig. 7 show maxima of modulation-index ratio of 1.4 and 2.1, at a modulating frequency of 3.0 Mc/s. At higher modulation frequencies, the modulation-index ratio falls monotonically. One or more of the following effects could account for the falling modulation-index ratio at high modulating frequencies: 1) the YIG or its associated circuit elements could introduce phase distortion yielding an envelope waveform different from that mentioned above, 2) sideband trimming by the YIG or its associated circuit elements could reduce the modulation index in a similar way as in AM radio receivers having insufficient IF bandwidth, and 3) insufficient postdetection bandwidth (about 20 Mc/s in our measuring equipment) would result in failure of the equipment to respond to the higher harmonics of the nonsinusoidal output envelope, and a reduction in the observed modulation index.

CONCLUSIONS

For applications where it is desired to remove amplitude variations from a carrier, the YIG filter will serve well for amplitude-modulation frequencies small enough that the carrier and the side frequencies are not limited selectively. At higher modulating frequencies, where the limiting is frequency selective, the YIG will not remove the amplitude variations. In the particular unit tested, the modulation index was reduced by a factor of two or more at all modulating frequencies below about 350 kc/s. At higher modulating frequencies, the reduction in modulation index was less than a factor of two, and, at modulating frequencies above about 750 kc/s, the modulation index was increased, rather than decreased, by the YIG limiter.

ACKNOWLEDGMENT

The authors are indebted to Dr. P. S. Carter, Jr., of Physical Electronics Labs., for many valuable discussions of ferrimagnetic devices, and to B. M. Ross, of Stanford Electronics Labs., for assistance in making the measurements.

REFERENCES

[1] K. Kotzebue, "Frequency selective limiting," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-10, pp. 516–520, November 1962.

[2] A. L. Berman, "Multiple-carrier behavior of a frequency-selective ferrite limiter," *IEEE Trans. on Communications Systems*, vol. CS-12, pp. 138–150, June 1964.

[3] R. L. Comstock and M. Loewy, "An X-band coincidence ferrimagnetic limiter using lithium ferrite," *Proc. IEEE*, vol. 51, pp. 1769–1770, December 1963.

[4] R. Arams, M. Grace, and S. Okwit, "Low-level garnet limiters," *Proc. IRE*, vol. 49, pp. 1308–1313; August 1961.

[5] F. J. Sansalone and E. G. Spencer, "Low-temperature microwave power limiter," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-9, pp. 272–273, May 1961.

[6] I. T. Ho, "Passive phase-distortionless parametric limiters," Electron Devices Lab., Stanford Electronics Labs., Stanford University, Calif. TR 157-2, April 24, 1961.

[7] W. W. Heinz, and S. Okwit, "A note on low power-level ferrite limiters," *Proc IEEE* (*Correspondence*), vol. 51, p. 367, February 1963.

[8] R. L. Comstock, W. A. Dean, and L. J. Varnerin, "A temperature-stabilized microwave power limiter for communication satellite use," *Proc. IEEE* vol. 50, pp. 470–471, April 1962.

[9] K. L. Kotzebue, "New trends in low-power ferrite limiters," *Electronics*, vol. 35, pp. 40–43, December 1962.