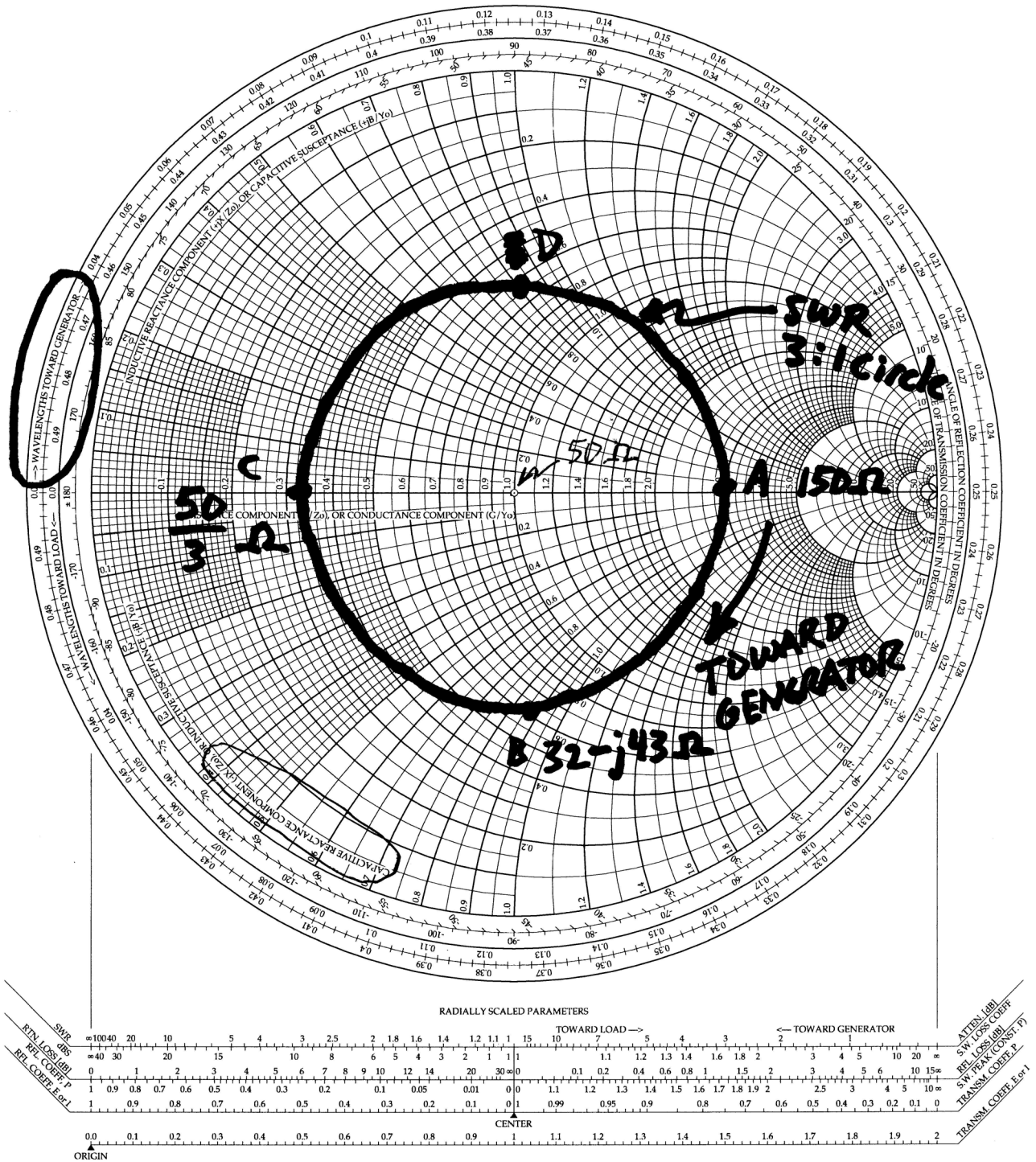


REFLECTIONS 4

FINAL EXAMPLE: MISMATCHED RESONANT 150 OHM ANTENNA



BEGINNING

We start with a 150 ohm resonant (no reactance) antenna, representing a 3:1 SWR. This could be made easily by shifting the feedpoint of a normal dipole a bit off from the center until we find the point of 150 ohms. It will be somewhere between the middle (50 ohms) and the point at 1/3 of length (200 ohms)

We feed this with 50 ohm coax, so all numbers of the "normalized" Smith Chart, we're going to multiply by FIFTY to get real impedances. Point A is our antenna, at 150 ohms purely resistive. The transmitter is many wavelengths away from the antenna.....not shown on our curve today.

To make things easy, lets assume we feed this transmission line back at the transmitter end with a perfect antenna tuner that automatically converts to 50 ohms so our transmitter is happy and then lets assume we are transmitting 150 watts.

FOR RESISTIVE IMPEDANCES (current in phase with voltage)

$$P = V * I$$

$$R = V / I$$

$$P = V * V / R \quad \text{or} \quad V = \text{square root of } (P * R)$$

$$P = I * I * R \quad \text{or} \quad I = \text{square root of } (P / R)$$

Lets use these forumulas to find the voltage and current coming out of the transmitter making 150 watts into 50 ohms of the antenna tuner.

Location	Power	Impedance	Voltage	Current
Transmitter (not shown on the curve)	150	50	86.6 volts	1.73

To check, we divide the voltage (86.6) by the current (1.73) and sure enough,, it is 50 ohms!
And if we multiply 86.6 volts x 1.73 A we get just what we expect: 150 watts.

POINT A: MAXIMUM VOLTAGE

At the ANTENNA end, (Point A) we are at 150 ohms and we can even intuitively see that 150 volts and 1 Amp works to give us both the right impedance and the right power (we set it up to work that way....) This is the point of maximum impedance, and thus the point at which we will see the highest voltage and the lowest current.

Location	Power	Impedance ohms	Voltage (RMS volts)	Current (RMS amps)
Transmitter (not shown on the curve)	150W	50 (resistive)	86.6 volts	1.73
Antenna = A	150W	150 (resistive)	150	1

POINT C: MAXIMUM CURRENT

Now step back exactly 1/4 wavelength from the antenna, on this mismatched 50 ohm coax, and we are at point C, where the impedance on the transmission line is purely resistive, and $50/3$ ohms = 16.6 ohms. THIS IS THE POINT WHERE WE WILL HAVE MINIMUM VOLTAGE AND MAXIMUM CURRENT BECAUSE THE IMPEDANCE IS AT A MINIMUM.

Using our formulas we can easily find that

Location	Power	Impedance ohms	Voltage (RMS volts)	Current (RMS amps)
Transmitter (not shown on the curve)	150W	50 (resistive)	86.6 volts	1.73
Antenna = A	150W	150 (resistive)	150	1
C 1/4 wavelength back from antenna	150 W	16.6 ohms resistive	50 volts	3 Amps

And sure enough, if we divide 50 volts by 3 Amps, we get 16.7 ohms, so it all checks out.

POINT B: 1/8 WAVE BACK FROM LOAD

Point B, is 45degrees (1/8 wavelength) back from the mismatched load. The impedance there is 32-j43 ohms (which can be found by multiplying FIFTY OHMS times the coordinates of resistive and reactive components at that location, approximately 0.63 resistive, and 0.86 reactive (capacitive because we are in the bottom half of the chart))

This 32-j43 ohms can be viewed as a magnitude and a phase. The phase will tell us the offset between voltage and current (by ohms law). Converting to polar can be done using some trig, or a calculator, but it is easier to use an online calculator such as:

<https://www.intmath.com/complex-numbers/convert-polar-rectangular-interactive.php>

Entering 32-j43 we get 54 complex ohms at 307 degrees (which can also be viewed as 54 complex ohms at -53 degrees).

$$Z = E / I = 54 \text{ ohms lagging } 53 \text{ degrees}$$

So the voltage here is 53 degrees out of phase with the current.

Real Power is Apparent Power (out of phase V multiplied by out of phase I) times cosine of the phase difference.

$$\text{Real P} = V * I * \text{cosine}(\text{phase offset}).$$

Cosine of 53 degrees can be found with a calculator as 0.60. Therefore....

$$\text{Real Power} = V * I * 0.6$$

So the "apparent power" (simple $V * I$, ignoring the phase offset) is $150/0.6$ or 250 watts!

The voltage here is found using that apparent power and our resistive formulae above:
square root of $250 * 54$ impedance ohms gives 116 volts

The current here is found using that apparent power and our resistive formulae above;
square root of $(250/54) = 2.15$ amps

And the real power is $V * I * \text{cosine (phase)}$

$$= 116 * 2.15 * 0.6 = 150 \text{ watts just like it should be!}$$

Location	Power	Impedance ohms	Voltage (RMS volts)	Current (RMS amps)
Transmitter (not shown on the curve)	150W	50 (resistive)	86.6 volts	1.73
Antenna = A	150W	150 (resistive)	150	1
C 1/4 wavelength back from antenna	150 W	16.6 ohms resistive	50 volts	3 Amps
B 1/8 wavelength back from antenna	150 Watts real power	54 complex lagging 53 degrees (power factor $\cos \theta = 0.6$)	116 volts rms	2.15 Amps (out of phase by 53 degrees with the voltage)

SWR = ratio of maximum voltage to minimum voltage = $150 / 50 = 3$ as expected

SWR = ratio of maximum current to minimum current = $3 / 1 = 3$ as expected.

EXPLAINING WHY SWR 3:1 is always half way up an SWR meter

Normal SWR meters made with parallel transmission lines (directional coupler) measure the ratio of the forward voltage to the reverse voltage. The SWR of 3 is always half way up the scale.

The REFLECTION COEFFICIENT for a 3:1 SWR for voltage or current is 0.5 (which is why the 3:1 mark on a SWR bridge is when the reflected voltage is half way up the scale to the forward voltage -- wow, now you understand how the SWR bridge's markings are figured out!

The REFLECTION COEFFICIENT of a 3:1 SWR for POWER is 0.25, so the reflected power will appear to be 1/4 of the forward power.

Forward power minus reflected power = real power = 150 watts
Forward power minus (1/4 forward) = 0.75 forward = 150 watts
or....apparent forward power = 200 watts. if this were a fifty ohm system
apparent reflected power = 50 watts if this were a fifty ohm system
True power = 150 watts -- all the time.

Lets look at those powers just at one point -- the 150 ohm (mismatched) antenna end:

(Remember there we have a forward and reverse wave from our first lesson on Reflections)

Apparent forward power 200 watts means on 50 ohms, $V_{\text{forward}} = 100$ volts, $I_{\text{forward}} = 2$ Amps
Apparent reverse power 50 watts means on 50 ohms, $V_{\text{reverse}} = 50$ volts; $I_{\text{reverse}} = 1$ amps

The voltages add to give the actual voltage at the antenna $100 + 50 = 150$
The currents are in opposite directions, so they subtract giving a total current of 1
and we get the actual 150 volts, 1 Amps, which fits with 150 ohms and 150 watts.

It all checks out.

STANDING WAVES OF VOLTAGE AND CURRENT

The transmission line can be many many wavelengths long, and every 1/4 wavelength along the line we will go from a voltage maximum to a voltage minimum and so on. These are the STANDING WAVES which if we could get a RMS voltage meter across, and an RMS ammeter inserted into, we would measure as we went along the transmission line.